



MANIPUR UNIVERSITY

Learning Outcomes-based Curriculum Framework (LOCF)

Semester Scheme with Multiple Entry and Exit Options for
Undergraduate Courses



Syllabus for Mathematics (I - VIII Semesters)

2022-23 onwards

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Syllabus for Bachelor of Science/Arts in Mathematics

Name of the Degree /Program: Bachelor of Science/ Bachelor of Arts

Discipline Course: Mathematics

Starting year of implementation: 2022-2023

Programme Outcomes (PO): By the end of the program the students will be able to gain the following skills.

PO1	Disciplinary knowledge: A bachelor degree in Mathematics is the culmination of in-depth knowledge of Algebra, Calculus, Geometry, Real analysis, Differential equations and several other branches of pure and applied mathematics, This also leads to the study of relevant areas such as computer science and other disciplines.
PO2	Communication Skills: Ability to communicate the various mathematical concepts effectively using a variety of examples mostly having real-life applications and their geometric visualization. The skills and knowledge gained in this programme will lead to proficiency in analytical reasoning which can be used to express thoughts and views in mathematically or logically correct statements.
PO3	Critical thinking and analytical reasoning: The students undergoing this programme acquire the ability of critical thinking and logical reasoning and will apply in formulating or generalizing specific hypothesis, conclusion. The learner will be able to recognize and distinguish the various aspects of real life problems.
PO4	Problem solving: The Mathematical knowledge gained by the student through this programme develops an ability to solve the problems, identify and define appropriate computing requirements for its solutions. This programme will enhance the overall development.
PO5	Research related skills: After the completion of this programme, the student will develop the capability of inquiring about appropriate questions relating to the Mathematical concepts, arguments. He/she will be able to define problems, formulate hypothesis, proofs, write the results obtained clearly.
PO6	Information/ digital literacy: The completion of this programme will enable the learner to use appropriate softwares to solve the system of algebraic and differential equations.
PO7	Self-directed learning: The student after the completion of the programme will be able to work independently, make an in-depth search of various areas of Mathematics and resources for self-learning in order to enhance knowledge in mathematics.
PO8	Moral and ethical awareness/reasoning: The student after the completion of the course will develop an ability to identify unethical behaviour such as fabrication, falsification or misinterpretation of data and adopting objectives, unbiased and truthful actions in all aspects of life in general and Mathematical studies in particular.
PO9	Lifelong learning: This programme provides self-directed learning and lifelong learning skills. With these skills, the learner will be able to think independently, and improve personal development.

Assessment

Weightage for the Assessments (in percentage)

Type of Course	Formative Assessment(I.A)	Summative Assessment (S.A)
Theory	25%	75%
Projects	25%	75%
Experimental Learning (Internship etc.)		

Course Structure

Model A

(A) Bachelor's Certificate in Mathematics (Level 5)

Semester	Discipline specific Core (DSC)	Discipline Specific Elective (DSE)	Generic Elective Course (GEC), (To be selected from GEC listings of other disciplines) (Credit)	Ability Enhancement Compulsory Courses (AECC)	Skill Enhancement Course (SEC) Select any one among (a)/(b)	Value Addition Courses (VAC) (To be selected from VAC listings of other disciplines). (Credits)	Semester Credit
I	MAT 501C (6)			AECC 1-- (4) English/MIL	MAT 501S (a)/(b) (4)	VAC 1 (2)	24
	MAT 502C (6)					VAC 2 (2)	
II	MAT 503C (6)			AECC 2-- (4) Environmental Science	MAT 502S (a)/(b) (4)	VAC 3 (2)	24
	MAT 504C(6)					VAC 4 (2)	

Award of Certificate in Mathematics (after 1st Year : minimum 46 (four-six) Credits)

(B) Bachelor's Diploma in Mathematics (Level 6)

Semester	Discipline specific Core (DSC)	Discipline Specific Elective (DSE)	Generic Elective Course (GEC), (Credit)	Ability Enhancement Compulsory Courses (AECC)	Skill Enhancement Course (SEC)	Value Addition Courses (VAC) (Credits)	Semester Credit
III	MAT 605C (6)		MAT 601G (6)			VAC 5 (2)	26
	MAT 606C (6)						
	MAT 607C (6)						
IV	MAT 608C (6)		MAT 602G (6)			VAC 6 (2)	26
	MAT 609C (6)						
	MAT 610C (6)						

Award of Diploma in Mathematics (after 2nd Year: minimum 96 (nine six) Credits)

(C) Bachelor's Degree in Mathematics (Level 7)

Semester	Discipline specific Core (DSC)	Discipline Specific Elective (DSE)	Generic Elective Course (GEC), (Credit)	Ability Enhancement Compulsory Courses (AECC)	Skill Enhancement	Value Addition Courses (VAC) (Credits)	Semester Credit
V	MAT 711C (6)	MAT 701D (6)	MAT 703G (6)			VAC 7 (2)	26
	MAT 712C (6)						
VI	MAT 713C (6)	MAT 702D (6)	MAT 704G (6)			VAC 8 (2)	26
	MAT 714C (6)						

Award of BSc degree in Mathematics (after 3rd Year: minimum 140 (one four zero) Credits)

(D) Bachelor's (Hons) Degree (Level 8)

Semester	Discipline specific Core (DSC)	Discipline Specific Elective (DSE)	Generic Elective Course (GEC), (Credit)	Ability Enhancement Compulsory Courses (AECC)	Skill Enhancement Course (SEC)	Value Addition Courses (VAC) (Credits)	Semester Credit
VII	MAT 815C (6)	MAT 803D (6)	MAT 805G (6)				24
	MAT 816C (6)						
VIII	MAT 817C (6)	MAT 804D (6) Research project/ Dissertation/ Internship	MAT 806G (6)				24
	MAT 818C (6)						

Award of B A/ B Sc degree with honours in Mathematics on completion of course equal to a minimum of 182 (one eight two) credits.

Bachelor of Science or Bachelor of Arts with Mathematics as Major & BA/BSc (Hons) Mathematics.

Course Structure

SEMESTER-WISE DISTRIBUTION OF COURSES

A. Discipline Specific Core (DSC) Courses:

All the courses have 6 credits with 5 (FIVE) credits of theory and 1(ONE) credit of tutorial

Sl. No.	CC Paper Code	Semester	Course Name
1.	MAT 501C	I	Calculus
2.	MAT 502C	I	Algebra
3.	MAT 503C	II	Real Analysis
4.	MAT 504C	II	Differential Equations
5.	MAT 605C	III	Theory of Real Functions
6.	MAT 606C	III	Group Theory
7.	MAT 607C	III	Multivariate Calculus
8.	MAT 608C	IV	Partial Differential Equations
9.	MAT 609C	IV	Riemann Integration
10.	MAT 610C	IV	Numerical Analysis
11.	MAT 711C	V	Metric Spaces
12.	MAT 712C	V	Mechanics
13.	MAT 713C	VI	Complex Analysis
14.	MAT 714C	VI	Ring Theory and Linear Algebra
15.	MAT 815C	VII	Abstract Algebra
16.	MAT 816C	VII	Advanced Real Analysis
17.	MAT 817C	VIII	Topology
18.	MAT 818C	VIII	Ordinary Differential Equations

B. Discipline Specific Electives (DSE):

All the courses have 6 credits with 4 credits of theory and 2 credits of practical or 5 credits of theory and 1 credit of tutorials.

Sl. No.	DSE Paper Code	Semester	DSE Name
1.	MAT 701D	V	(a) Advanced Group Theory, (b) Mathematical Modeling, (c) Integral Transforms
2.	MAT 702D	VI	(a) Special Theory of Relativity & Tensor, (b) Linear Programming & its applications, (c) Probability Theory & Statistics
3.	MAT 803D	VII	(a) Advanced Complex Analysis (b) Functional Analysis (c) Graph Theory
4.	MAT 804D	VIII	(a) Advanced Partial Differential Equations, (b) Fixed point theory, (c) Cryptography

C. Skill Enhancement Courses (SEC):

All courses have 4 credits with 2 credits of theory and 2 credits of Practical/Tutorials/Projects and Field Work to be decided by the College.

Sl No.	DSE Paper Code	Semester	SEC Name
1.	MAT 501S (a)	I	LaTeX
2.	MAT 501S (b)	I	Computational Mathematics Laboratory
3.	MAT 502S (a)	II	Python Programming
4.	MAT 502S (b)	II	Computer Algebra Systems and Related Software

D. Ability Enhancement Compulsory Courses:

All the courses have 4 credits including Theory/Practicals/Projects.

Sl No.	AECC Paper Code	Semester	AECC Name
1.	AECC 1--	I	English/MIL
2.	AECC 2--	II	Environmental Science

E. Value Addition Courses:

Sl.No	VAC Paper Code	Semester	VAC Name
1.	VAC 1	I	Yoga
2.	VAC 2	I	Sports
3.	VAC 3	II	Culture
4.	VAC 4	II	Health Care
5.	VAC 5	III	NCC
6.	VAC 6	IV	Ethics
7.	VAC 7	V	NSS
8.	VAC 8	VI	History of Science

*The list is only indicative and it depends on the available options in the institution/college.

F. Generic Elective Courses:

All the courses have 6 credits with 5 credits of theory and 1 credit of tutorial. These courses are meant for students of other departments/disciplines.

Sl. No.	GEC Paper Code	Semester	GEC Name
1.	MAT 601G	III	Quantitative Aptitude
2.	MAT 602G	IV	Basic Tools of Mathematics
3.	MAT 703G	V	Recreational Mathematics
4.	MAT 704G	VI	Discrete Mathematics
5.	MAT 805G	VII	Analytical Geometry and Theory of Equations
6.	MAT 806G	VIII	Numerical Methods

Contents of Courses for B.A./B.Sc. degree with honours in Mathematics Model B

Semester	Course Code		Credit	Paper Title	Marks		Remark
					S.A	I.A.	
I	MAT 501C	Theory	6	Calculus	75	25	Approved with Syllabus on 09/05/2022
	MAT 502C	Theory	6	Algebra	75	25	
	MAT 501S	Theory & practical	4	(a) Latex, (b) Computational Mathematics Laboratory	37.5 37.5	12.5 12.5	
II	MAT 503C	Theory	6	Real Analysis	75	25	
	MAT 504C	Theory	6	Differential Equations	75	25	
	MAT 502S	Theory& Practical	4	(a) Python Programming, (b) Computer Algebra Systems and related Software	37.5 37.5	12.5 12.5	
Exit Option with Certificate							
III	MAT 605C	Theory	6	Theory of Real Functions	75	25	Approved with Syllabus on 09/05/2022
	MAT 606C	Theory	6	Group Theory	75	25	
	MAT 607C	Theory	6	Multivariate Calculus	75	25	
	MAT 601G	Theory	6	Quantitative Aptitude	75	25	
IV	MAT 608C	Theory	6	Partial Differential Equations	75	25	
	MAT 609C	Theory	6	Riemann Integration	75	25	
	MAT 610C	Theory	6	Numerical Analysis	75	25	
	MAT 602G	Theory	6	Basic Tools of Mathematics	75	25	
Exit Option with Diploma							
V	MAT 711C	Theory	6	Metric Spaces	75	25	Approved with syllabus on 21/09/2023
	MAT 712C	Theory	6	Mechanics	75	25	
	MAT 701D	Theory	6	(a) Advanced Group Theory (b) Mathematical Modelling (c) Integral Transforms	75	25	
	MAT 703G	Theory	6	Recreational Mathematics	75	25	
VI	MAT 713C	Theory	6	Complex Analysis	75	25	
	MAT 714C	Theory	6	Ring Theory and Linear Algebra	75	25	
	MAT 702D	Theory	6	(a) Special Theory of Relativity & Tensor (b) Linear Programming & its applications (c) Probability Theory and Statistics	75	25	
	MAT 704G	Theory	6	Discrete Mathematics	75	25	
Exit Option with Bachelor of Arts, B.A./Bachelor of Science, B.Sc.							
VII	MAT 815C	Theory	6	Abstract Algebra	75	25	Approved with syllabus on 01/12/2023
	MAT 816C	Theory	6	Advanced Real Analysis	75	25	
	MAT 803D	Theory	6	(a) Advanced Complex Analysis (b) Functional Analysis (c) Graph Theory	75	25	
	MAT 805G	Theory	6	Analytic Geometry and Theory of Equations	75	25	
VIII	MAT 817C	Theory	6	Topology	75	25	
	MAT 818C	Theory	6	Ordinary Differential Equations	75	25	
	MAT 804D	Theory	6	(a) Advanced Partial Differential Equations (b) Fixed Point Theory, (c) Cryptography	75	25	
	MAT 806G	Theory	6	Numerical Methods	75	25	

Award of Bachelor of Arts / Science (BA/BSc) Honours in Mathematics

Discipline Specific Core Courses

SEMESTER I

MAT 501C : Calculus

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week) 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The primary objective of this course is to introduce the basic tools of calculus and geometric properties of different conic sections which are helpful in understanding their applications in planetary motion, design of telescope and to the real-world problems. Also, to carry out the hand on sessions in computer lab to have a deep conceptual understanding of the above tools to widen the horizon of students' self-experience.

Course Learning Outcomes: After completion of the course, a student will be able to:

- i) sketch curves in a plane in the different coordinate systems of reference.
- ii) understand the Calculus of vector valued functions.
- iii) apply calculus to develop basic principles of planetary motions.

Unit 1: Derivatives for Curve sketching

(35 marks, 5 weeks)

First and second derivative tests for Extreme Values of Functions, Concavity and Curve Sketching, Limits to infinity and infinite limits, Indeterminate Forms and L'Hôpital's Rule, Asymptotes, Higher order derivatives, Leibniz rule.

Unit 2: Curve tracing in polar Co-ordinates

(30 marks, 4 weeks)

Parametric representation of curves, Polar Coordinates, Tracing of curves in Polar Coordinates, Graphing Polar Coordinate Equations, Areas and Lengths in Polar Coordinates, Classification of conics in Polar Coordinates.

Unit 3: Vector Calculus and its applications

(35 marks, 5 weeks)

Vector-valued functions and their graphs, Limits and continuity of vector functions, Differentiation and integration of vector functions, Projectile motion, Unit tangent, Normal and binormal vectors, Curvature, Kepler's Second Law (Equal Area Law).

References:

1. **Thomas, Jr. George B., Weir, Maurice D., & Hass, Joel** (2014). *Thomas' Calculus* (13th ed.) Pearson Education, Delhi. Indian Reprint 2017.
2. **B. C. Das, B. N. Mukherjee.** *Differential Calculus* (55th Edition), U.N. Dhur & Sons Private Ltd., Kolkata (2015).

Teaching plan (MAT 501C Calculus):

- Week 1:** First and second derivative tests for Extreme Values of Functions; [1] Chapter 4 (Section 4.3).
Week 2: Concavity and Curve Sketching; [1] Chapter 4 (Section 4.4).
Week 3: Limits to infinity and infinite limits; [1] Chapter 2 (Section 2.6).
Week 4: Indeterminate forms and L'Hospital's Rule; [1] Chapter 4 (Section 4.5), Asymptotes; [1] Chapter 2 (Section 2.6).
Week 5: Higher order derivatives; [1] Chapter 3 (Section 3.7), Leibniz rule; [1] Chapter 3 (Section 3.11).
Week 6: Parametric representation of curves; [1] Chapter 11 (Section 11.1 and 11.2),
Week 7: Polar Coordinates, Tracing of curves in Polar Coordinates; [1] Chapter 11 (Section 11.3).
Week 8: Graphing Polar Coordinates Equations; [1] Chapter 11 (Section 11.4), Areas and Lengths in Polar Coordinates; [1] Chapter 11 (Section 11.5).
Week 9: Classification of Conics in Polar Coordinates; [1] Chapter 11 (Section 11.6 and 11.7).
Week 10: Vector valued functions and their graphs, Limits and Continuity of vector functions; [1] Chapter 13 (Section 13.1).
Week 11: Differentiation and integration of vector functions; [1] Chapter 13 (Section 13.1).
Week 12: Projectile motion; [1] Chapter 13 (Section 13.2), Unit tangent; [1] Chapter 13 (Section 13.3).
Week 13: Normal and binormal vectors; [1] Chapter 13 (Section 13.3).
Week 14: Curvature; [1] Chapter 13 (Section 13.4 and 13.5), Kepler's Second Law (Equal Area Law); [1] Chapter 13 (Section 13.6).

MAT 502C : Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week) 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The primary objective of this course is to introduce the basic tools of theory of equations, complex numbers, number theory and matrices to understand their linkage to the real-world problems.

Course Learning Outcomes: After completion of the course, a student will be able to

- Employ De Moivre's theorem in a number of applications to solve numerical problems;
- Apply Euclid's algorithm and backwards substitution to find the greatest common divisor;
- Recognize consistent and inconsistent systems of linear equations by using rank.

Unit 1: Theory of Equations

(35 marks, 5 weeks)

Polynomial functions, Division algorithm, Synthetic division, Remainder Theorem, Factor Theorem, Polynomial equations, Relation between roots and Coefficients of a polynomial equation, Symmetric function of the roots of an equation, sum of powers of the roots, Solution of cubic and biquadratic equations, De Moivre's Theorem for integer and fractional indices.

Unit 2: Relation, functions and Basic Number Theory

(35 marks, 5 weeks)

Binary relations, Partial order relation, Equivalence relations, Functions, Inverses and composition, One to one correspondence and Cardinality of a set, Division Algorithm, Divisibility and the Euclidean Algorithm, Prime Numbers, Congruences and applications, Principles of Mathematical induction.

Unit 3: Matrices

(30 marks, 4 weeks)

Rank of a matrix, Rank and elementary operations, Row reduction and echelon forms, System of linear equations, Solution of the matrix equation $AX=B$, Solution sets of linear systems, linear independence, Eigenvectors and Eigen values, The Characteristic equation and Cayley- Hamilton Theorem.

References:

1. **Goodaire, Edgar G & Parmentor, Michael M** (2005); *Discrete Mathematics with Graph Theory* (3rd Ed.) Pearson Education Pvt. Ltd., Indian Reprint 2015
2. **MK Singal, Asha Rani Singal**, (2020); *Algebra* (31st Ed) R Chand &Co, New Delhi.
3. **Chandrika Prasad**, (1963). *Text Book on Algebra and Theory of Equations* Pothishala Pvt. Ltd.

Additional Readings:

1. **Kolman, Bernard, & Hill, David R.** (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.
2. **Lay, David C., Lay, Steven R., & McDonald, Judi J.** (2016). *Linear Algebra and its Applications* (5th ed.). Pearson Education.
3. **Andrilli, Stephen, & Hecker, David** (2016). *Elementary Linear Algebra* (5th ed.). Academic Press, Elsevier India Private Limited.
4. **Burton, David M.** (2007). *Elementary Number Theory* (7th ed.). Tata Mc-Graw Hill Edition, Indian Reprint.

Teaching plan (MAT 502C : Algebra):

Week 1: Polynomial functions, Division Algorithm, Synthetic division; [2] Chapter 3 (Section 3.2, 3.3 &3.4).

Week 2:Remainder Theorem, Factor Theorem;[1] Chapter 4 (Section 4.1),

Week 3:Polynomial equations, Relations between roots and Co-efficients of a polynomial equation; [2] Chapter 3 (Section 3.6 &3.7).

Week 4:Symmetric functions of the roots of an equation, Sum of the powers of the roots; [2] Chapter 3 (Section 3.10 &3.10), Solutions of cubic and biquadratic equations; [3]Chapter 13(Section 13.2, 13.3, 13.6, 13.7)

Week 5:DeMoivre's Theorem for integer and fractional indices; [2] Chapter 4 (Section 4.1).

Week 6:Binary relations, Partial order relation, Equivalence relations; [1] Chapter 2 (Section 2.3 &2.4).

Week 7:Functions,Domain, Range, One-One, Onto, Inverses and composition, One to One correspondence and Cardinality of a set; [1] Chapter 3 (Section 3.1, 3.2 &3.3).

Week 8:Division Algorithm, Divisibility and The Euclidean Algorithm;[1] Chapter 4 (Section 4.2).

Week 9:Prime Numbers, Congruences and applications; [1] Chapter 4 (Section 4.3 & 4.4).

Week 10:Principle of Mathematical Induction; [1] Chapter 5 (Section 5.1).

Week 11:Rank of a matrix, Rank and elementary operations; [2] Chapter 6 (Section 6.2 & 6.3).

Week 12:System of linear equations, Solution of the matrix equation $AX=B$; [2] Chapter 7 (Section 7.2 &7.3)

Week 13:Solution sets of linear systems, linear independence; [2] Chapter 6 (Section 6.4), Eigenvectors and Eigen values; [2] Chapter 8 (Section 8.2).

Week 14:The Characteristic equation and Cayley-Hamilton Theorem; [2] Chapter 8 (Section 8.4).

SEMESTER II

MAT 503C : Real Analysis

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The course will develop a deep and rigorous understanding of real lines of defining terms to prove the results of convergence and divergence of sequences and series of real numbers. These concepts has wide range of applications in real-life scenario.

Course Learning Outcomes: This course will enable the students to:

- i) Understand many properties of the real line \mathbb{R} and learn to define sequence in terms of functions from to a subset of \mathbb{R} .
- ii) Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate their limit superior, limit inferior, and the limit of a bounded sequence.
- iii) Apply the ratio, root, alternating series and limit comparison tests for convergence and absolute convergence of an infinite series of real numbers.

Unit 1: Real Number System and its properties

(30 Marks, 4 Weeks)

Algebraic and order properties of \mathbb{R} , Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of \mathbb{R} , the completeness property of \mathbb{R} , Archimedean property, Density of rational numbers in \mathbb{R} ; Definition and types of intervals, Nested intervals property; Neighbourhood of a point in \mathbb{R} , Open and closed sets in \mathbb{R} .

Unit 2: Sequences in \mathbb{R}

(35 Marks, 5 Weeks)

Convergent sequence, Limit of a sequence, Bounded sequence, Limit theorems, Monotone sequences, Monotone convergence theorem, Sub-sequences, Bolzano-Weierstrass theorem for sequences, Limit superior and limit inferior for bounded sequence, Cauchy sequence, Cauchy's convergence criterion.

Unit 3: Infinite Series

(35 Marks, 5 Weeks)

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence; Tests for convergence of positive term series: Integral test, Basic comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's n^{th} root test; Alternating series, Leibniz test, Absolute and conditional convergence.

References:

1. **Bartle, Robert G., & Sherbert, Donald R.** (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition. New Delhi.
2. **Ross, Kenneth A.** (2013). *Elementary Analysis: The theory of calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.
3. **Denlinger, Charles G.** (2011). *Elements of Real Analysis*. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Additional Readings:

1. **Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E.** (2010). *An Introduction to Analysis* (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
2. **Thomson, Brian S., Bruckner, Andrew. M., & Bruckner, Judith B.** (2001). *Elementary Real Analysis*. Prentice Hall.

Teaching Plan (MAT 503C: Real Analysis):

Weeks 1 and 2: Algebraic and order properties of \mathbb{R} . Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of \mathbb{R} .

[1] Chapter 2 [Sections 2.1, 2.2 (2.2.1 to 2.2.6), and 2.3 (2.3.1 to 2.3.5)]

Weeks 3 and 4: The completeness property of \mathbb{R} , Archimedean property, Density of rational number in \mathbb{R} ; Definition and types of intervals, Nested intervals property; Neighborhood of a point in \mathbb{R} , Open and closed sets in \mathbb{R} .

[1] Chapter 2 [Sections 2.3 (2.3.6), 2.4 (2.4.3 to 2.4.9), and 2.5 up to Theorem 2.5.3]

[1] Chapter 11 [Section 11.1 (11.1.1 to 11.1.3)]

Weeks 5 and 6: Convergent sequence, Sequences and their limits, Bounded sequence, Limit theorems.

[1] Chapter 3 (Sections 3.1 and 3.2)

Week 7: Monotone sequences, Monotone convergence theorem and applications.

[1] Chapter 3 (Section 3.3)

Week 8: Subsequences and statement of the Bolzano-Weierstrass theorem, Limit superior and limit inferior for bounded sequence of real numbers with illustrations only.

[1] Chapter 3 [Section 3.4 (3.4.1 to 3.4.12), except 3.4.4, 3.4.7, 3.4.9 and 3.4.11]

Week 9: Cauchy sequences of real numbers and Cauchy's convergence criterion.

[1] Chapter 3 [Section 3.5 (3.5.1 to 3.5.6)]

Week 10: Convergence and divergence of infinite series, Sequence of partial sums of infinite series, Necessary condition for convergence, Cauchy criterion for convergence of series.

[3] Chapter 8 (Section 8.1)

Weeks 11 and 12: Tests for convergence of positive term series: Integral test statement and convergence of p -series, Basic comparison test, Limit comparison test with applications, D'Alembert's ratio test and Cauchy's n th root test.

[3] Chapter 8 (Section 8.2 up to 8.2.19)

Weeks 13 and 14: Alternating series, Leibniz test, Absolute and conditional convergence.

[3] Chapter 8 [Section 8.3 (8.3.1 to 8.3.7)]

MAT 504C : Differential Equations

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70) **Examination:** 3 Hrs.

Course Objectives: The main objectives of this course are to introduce the students to the exciting world of Differential Equations, Mathematical Modeling and their applications.

Course Learning Outcomes: The course will enable the students to:

- i) Formulate Differential Equations for various Mathematical models.
- ii) Solve first order non-linear differential equation and linear differential equations of higher order using various techniques.
- iii) Apply these techniques to solve and analyze various mathematical models.

Unit 1: Differential Equations and Mathematical Modeling

(35 marks, 5 weeks)

Differential equations and mathematical models, Order and degree of a differential equations, Integrals as general and particular solutions, Exact differential equations and integrating factors of first order differential equations, Separable Equations, Homogeneous Equations, Reduction to homogeneous equations, Linear equations and Bernoulli Equation, Clairaut's Equation, Existence and Uniqueness of solution of initial and boundary value problems of first order ODE, singular solution of first order ODE.

Unit 2: Second and higher order differential Equations

(35 marks, 5 weeks)

General solution of homogeneous equation of second order, Principle of superposition for a homogeneous equation, Wronskian, its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, Method of undetermined coefficients, Method of variation of parameters, Applications of second order differential equations to mechanical vibration.

Unit 3: Analysis of Mathematical Models

(30 marks, 4 weeks)

Application of first order differential equations to acceleration-velocity model, Growth and Decay model. Introduction to compartmental models, Lake pollution model (with case study of Lake Burley Griffin), Drug Assimilation models, population models (with limited growth, exponential growth) Epidemic models.

References:

1. **Barnes, Belinda & Fulford, Glenn R.** (2015). *Mathematical Modelling with Case Studies, Using Maple and MATLAB* (3rd ed.). CRC Press, Taylor & Francis Group.
2. **Edwards, C. Henry, Penney, David E., & Calvis, David T.** (2015). *Differential Equation and Boundary Value Problems: Computing and Modeling* (5th ed.). Pearson Education.
3. **Ross, Shepley L.** (2004). *Differential Equations* (3rd ed.). John Wiley & Sons. India.

Teaching plan (MAT 504C : Differential Equations)

Week 1: Differential equations and mathematical models; [2] Chapter 1 (Section 1.1), Order and degree of a differential equations; [3] Chapter 1 (Section 1.1)

Week 2: Integrals as general and particular solutions; [2] Chapter 1 (Section 1.2), Exact differential equations and integrating factors of first order differential equations; [3] Chapter 2 (Section 2.1)

Week 3: Separable equations, Homogeneous equations, Reduction to homogeneous equations; [3] Chapter 2 (Section 2.2).

Week 4: Linear equations and Bernoulli equation Clairaut's equation ; [3] Chapter 2 (Section 2.3)

Week 5: Existence and Uniqueness of solution of initial and boundary value problems of first order ODE; singular solution of the first order ODE [3] Chapter 1 (Section 1.3),

Week 6 & 7: General solution of homogeneous equation of second order, Principle of superposition for a homogeneous equation, Wronskian, its properties and applications; [2] Chapter 3 (Section 3.1).

Week 8: Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation; [2] Chapter 3 (Section 3.3).

Week 9: Method of undetermined coefficients, Method of variation of parameters; [2] Chapter 3 (Section 3.5).

Week 10: Applications of second order differential equations to mechanical vibration; [2] Chapter 3 (Section 3.6).

Week 11& 12: Application of first order differential equations to acceleration-velocity model; [2] Chapter 2 (Section 2.3), Growth and Decay model; [1] Chapter 2 (Section 2.2).

Week 13: Introduction to compartmental models; [1] Chapter 2 (Section 2.1), Lake pollution model (with case study of Lake Burley Griffin); [1] Chapter 2 (Section 2.5 & 2.6), Drug Assimilation models; [1] Chapter 2 (Section 2.7).

Week 14: Population models (with limited growth, exponential growth) Epidemic models; [2] Chapter 2 (Section 2.1) or [1] Chapter 3 (Section 3.1)

SEMESTER III

MAT 605C : Theory of Real Functions

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: It is a basic course on the study of real valued functions that would develop an analytical ability to have a more matured perspective of the key concepts of calculus, namely, limits, continuity, differentiability and their applications.

Course Learning Outcomes: This course will enable the students to learn:

- i) A rigorous approach of the concept of limit of a function.
- ii) About continuity and uniform continuity of functions defined on intervals.
- iii) The geometrical properties of continuous functions on closed and bounded intervals.
- iv) The applications of mean value theorem and Taylor's theorem.

Unit 1: Limits of Functions

(20 Marks, 3 Weeks)

Limits of functions (ε - δ approach), Sequential criterion for limits, Divergence criteria, Limit theorems, One-sided limits, Infinite limits and limits at infinity.

Unit 2: Continuous Functions and their Properties

(35 Marks, 5 Weeks)

Continuous functions, Sequential criterion for continuity and discontinuity, Algebra of continuous functions, Properties of continuous functions on closed and bounded intervals; Uniform continuity, Non-uniform continuity criteria, Uniform continuity theorem.

Unit 3: Derivability and its Applications

(45 Marks, 6 Weeks)

Differentiability of a function, Algebra of differentiable functions, Carathéodory's theorem and chain rule; Relative extrema, Interior extremum theorem, Rolle's theorem, Mean-value theorem and its applications, Intermediate value property of derivatives - Darboux's theorem, Taylor polynomial, Taylor's theorem with Lagrange form of remainder, Application of Taylor's theorem in error estimation; Relative extrema, and to establish a criterion for convexity; Taylor's series expansions of e^x , $\sin x$ and $\cos x$

Reference:

1. **Bartle, Robert G., & Sherbert, Donald R.** (2015). *Introduction to Real Analysis* (4thed.). Wiley India Edition. New Delhi.

Additional Readings:

1. **Ghorpade, Sudhir R. & Limaye, B. V.** (2006). *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE). First Indian reprint.
2. **Mattuck, Arthur.** (1999). *Introduction to Analysis*, Prentice Hall.
3. **Ross, Kenneth A.** (2013). *Elementary Analysis: The theory of calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.
4. **Howard Anton, I. Bivens & Stephan Davis** (2016). *Calculus* (10th Edn.), Wiley, India.
5. **Gorakh Prasad** (2016). *Differential Calculus* (19th Edn.), Pothisala Pvt. Ltd.

MAT 606C : Group Theory

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of the course is to introduce the fundamental theory of groups and their homomorphisms. Symmetric groups and group of symmetries are also studied in detail. Fermat's Little theorem as a consequence of the Lagrange's theorem on finite groups.

Course Learning Outcomes: After completion of the course, a student will be able to

- i) Understand the basic concepts of groups and links with symmetric figures;
- ii) learn concepts of normal subgroups, cosets and quotient groups;
- iii) learn the concepts of group homomorphisms and isomorphisms.

Unit 1: Groups and elementary properties

(35 Marks, 5 Weeks)

Symmetries of a Square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups, cycle notation of permutations, properties of permutations, Elementary properties of groups, Permutations, Even and odd permutations.

Unit 2: Subgroups

(35 Marks, 5 Weeks)

Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a group, Cosets of a Group, Lagrange's theorem and consequences including Fermat's Little theorem, cyclic groups, Classification of subgroups of cyclic groups, Normal subgroups, Quotient Groups, alternating groups.

Unit 3: Group Homomorphisms

(30 Marks, 4 Weeks)

Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Properties of isomorphisms, First, Second and Third isomorphism theorems for groups, Cayley's theorem

Reference

1. **Gallian, Joseph. A.** (2013). *Contemporary Abstract Algebra* (8th ed.). Cengage Learning India Private Limited, Delhi. Fourth impression, 2015.
2. **I.N. Herstein,** (2006). *Topics in Algebra* (2nd Edn). Wiley India Pvt. Ltd.

Additional Reading:

1. **V.K. Khanna, SK Bhambri** (2017). *A course in Abstract Algebra* (5th Edn). Vikas Pub. House Pvt Ltd.
2. **Rotman, Joseph J.** (1995). *An Introduction to the Theory of Groups* (4th ed.). Springer Verlag, NY.
3. **Michael Artin** (2014). *Algebra* (2nd Edn.), Pearson.
4. **Nathan Jacobson** (2009). *Basic Algebra I* (2nd Edn.), Dover Publication.
5. **John B. Fraleigh** (2007). *A First Course in Abstract Algebra* (7th Edn.) Pearson.
6. **I. S. Luthar & I.B.S. Passi** (2013). *Algebra Volume I Group*, Narosa.

Teaching Plan (MAT 606C: Group Theory):

Week 1: Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices). [1] Chapter 1.

Week 2: Definition and examples of groups, Elementary properties of groups. [1] Chapter 2.

Week 3: Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a Group, Product of two subgroups. [1] Chapter 3.

Weeks 4 and 5: Properties of cyclic groups. Classification of subgroups of cyclic groups. [1] Chapter 4

Weeks 6 and 7: Cycle notation for permutations, Properties of permutations, Even and odd permutations, [1] Chapter 5 (up to Page 110).

Weeks 8 and 9: Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem. [1] Chapter 7 (up to Example 6, Page 150).

Week 10: Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups. [1] Chapters 9 (Theorem 9.1, 9.2, 9.3 and 9.5, and Examples 1 to 12), Alternating group, [1] Chapter 5 (up to Page 110).

Weeks 11 and 12: Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem. [1] Chapter 10 (Theorems 10.1 and 10.2, Examples 1 to 11). [1] Chapter 6 (Theorem 6.1, and Examples 1 to 8).

Weeks 13 and 14: Properties of isomorphisms, First, Second and Third isomorphism theorems. [1] Chapter 6 (Theorems 6.2 and 6.3), Chapter 10 (Theorems 10.3, 10.4, Examples 12 to 14, and Exercises 41 and 42 for second and third isomorphism theorems for groups).

MAT 607C : Multivariate Calculus

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs. Theory) **Examination:** 3 Hrs.

Course Objectives: To understand the extension of the studies of single variable differential and integral calculus to functions of two or more independent variables. Also, the emphasis will be on the use of Computer Algebra Systems by which these concepts may be analyzed and visualized to have a better understanding.

Course Learning Outcomes: This course will enable the students to learn:

- i) The conceptual variations when advancing in calculus from one variable to multivariable discussions.
- ii) Inter-relationship amongst the line integral, double and triple integral formulations.
- iii) Applications of multi variable calculus tools in physics, economics, optimization, and understanding the architecture of curves and surfaces in plane and space etc.

Unit 1: Calculus of Functions of Several Variables and Properties of Vector Field (40 Marks, 6 weeks)

Functions of several variables, Level curves and surfaces, Limits and continuity, Partial differentiation, Higher order partial derivative, Tangent planes, Total differential and differentiability, Chain rule, Directional derivatives, The gradient, Maximal and normal property of the gradient, Tangent planes and normal lines, Extrema of functions of two variables, Method of Lagrange multipliers, Constrained optimization problems; Definition of vector field, Divergence and curl.

Unit 2: Double and Triple Integrals – (30 Marks, 4 Weeks)

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelepiped and solid regions, Volume by triple integrals, triple integration in cylindrical and spherical coordinates, Jacobians (Without Proof), Change of variables in double and triple integrals.

Unit 3: Green's, Stokes' and Gauss Divergence Theorem – (30 Marks, 4 Weeks)

Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral; Surface integrals, Stokes' theorem, The Gauss divergence theorem.

References:

1. **Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J.** (2007). *Calculus* (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.
2. **Marsden, J. E., Tromba, A., & Weinstein, A.** (2004). *Basic Multivariable Calculus*. Springer (SIE). First Indian Reprint.

Additional Reading:

1. **Steward James** (2012). *Multivariable Calculus* (7th Edn.), Books/Cole, Cengage.
2. **George B, Thomas Jr., Joel Hass, Christopher Heil & Maurice D. Weir** (2018). *Thomas' Calculus* (14th Edn.) Pearson.

Teaching Plan (MAT 607C: Multivariate Calculus):

Week 1: Definition of functions of several variables, Graphs of functions of two variables-Level curves and surfaces, limits and continuity of functions of two variables.

[1] Chapter 11(Sections 11.1 and 11.2)

Week 2: Partial differentiation, and partial derivative as slope and rate, Higher order partial derivatives, Tangent Planes, incremental approximation, Total differential.

[1] Chapter 11 (Sections 11.3 and 11.4)

Week 3: Differentiability, Chain rule for one parameter, Two and three independent parameters.

[1] Chapter 11 (Section 11.4 and 11.5)

Week 4: Directional derivatives, The gradient, Maximal and normal property of the gradient, Tangent planes and normal lines.

[1] Chapter 11 (Section 11.6)

Week 5: First and second partial derivative tests for relative extrema of functions of two variables, and absolute extrema of continuous functions.

[1] Chapter 11(section 11.7 (upto page 605))

Week 6: Lagrange multipliers method for optimization problems with one constraint, Definition of vector field, Divergence and curl.

[1] Chapter 11 [Section 11.8 (pages 610-714C) Chapter 13 (section 13.1)

Week 7: Double integration over rectangular and nonrectangular regions.

[1] Chapter 12(Sections 12.3 and 12.4)

Week 8: Double integrals in polar coordinates, and triple integral over a parallelepiped.

[1] Chapter 12 (Sections 12.3 and 12.4)

Week 9: Triple integral over solid regions, Volume by triple integral, and triple integration in cylindrical coordinates.

[1] chapter 12 (Sections 12.4 and 12.5)

Week 10: Triple integration in spherical coordinates, Jacobian (Without Proof), Change of variables in double and triple integrals.

[1] Chapter 12(Sections 12.7 and 12.8 upto page 849)

Week 11: Line integrals and its properties, applications of line integrals: mass and work.

[1] Chapter 13 (Section 13.2)

Week 12: Fundamental theorem for line integrals, Conservative vector fields and path independence.

[1] Chapter 13 (Section 13.3)

Week 13: Green's theorem for simply connected region, area as a line integral, Definition of surface integrals.

[1] Chapter 13 (Sections 13.4 and 13.7)

Week 14: Stokes' theorem and the divergence theorem.

[1] Chapter 13 (Sections 13.6 and 13.7)

SEMESTER-IV

MAT 608C : Partial Differential Equations

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (56 Hrs. Theory) **Examination:** 3 Hrs.

Course Objectives: The main objectives of this course are to teach students to form and solve partial differential equations and use them in solving some physical problems.

Course Learning Outcomes: The course will enable the students to

- i. Formulate, classify and transform partial differential equations into canonical form
- ii. Solve linear and non-linear partial differential equations using various methods: and apply these methods in solving some physical problems.

Unit 1. First order PDE and Methods of Characteristics

(30 Marks, 4 Weeks)

Definitions & Basic concepts, Formation of PDE, classification and geometrical interpretation of first order partial differential equations (PDE), Method of characteristics and general solution of first order PDE, Lagrange and Charpit method, Cauchy's problems for first order PDE, Canonical form of first order PDE, Method of separation of variables for first order PDE

Unit 2. Classification of second order Linear PDE and Wave equations (35 Marks, 5 Weeks)

Classification of second order PDE, Reduction to canonical forms, Equations with constant coefficients, General solutions, Cauchy's Problem for second order PDE, Mathematical Modeling of vibrating string, vibrating membrane, Homogeneous wave equation, Initial boundary value problems, Non-homogeneous boundary conditions, Finite string with fixed ends, Non-homogeneous wave equation.

Unit 3. Methods of separation of Variables (35 Marks, 5 Weeks)

Methods of separation of Variables for second order PDE, vibrating string problems, Existence and uniqueness of solution of vibrating string problems, Heat conduction problem, Existence and uniqueness of solution of Heat conduction problems, General solution of higher order PDE with constant coefficient, Non-homogeneous Problems.

References:

1. **Myint-U, Tyn and Debnath, Lokenath.** (2007). *Linear Partial Differential Equation for Scientists and Engineers* (4thed). Springer, Third Indian Reprint.

Additional Readings:

1. **Sneddon, I. N.** (2006). *Elements of Partial Differential Equations*, Dover Publications. Indian Reprint.
2. **Stavroulakis, Ioannis P & Tersian, Stepan A.** (2004). *Partial Differential Equations: An Introduction with Mathematica and MAPLE* (2nd ed.). World Scientific.
3. **Kreyszig E.** (2011). *Advanced Engineering Mathematics* (10th Edn.) Wiley.
4. **Piaggio H.T.H** (2004). *An Elementary Treatise on Differential Equations and its application*, CBS Publication.

Teaching Plan (Theory of MAT 608C : Partial Differential Equations):

Week 1: Introduction, Classification, Construction of first order partial differential equations (PDE).

[1] Chapter 2 (Sections 2.1 to 2.3)

Week 2: Method of characteristics and general solution of first order PDE.

[1] Chapter 2 (Sections 2.4 and 2.5)

Week 3: Canonical form of first order PDE, Method of separation of variables for first order PDE.

[1] Chapter 2 (Sections 2.6 and 2.7)

Week 4: The vibrating string, Vibrating membrane, Gravitational potential, Conservation laws.

[1] Chapter 3 (Sections 3.1 to 3.3, 3.5, and 3.6)

Weeks 5 and 6: Reduction to canonical forms, Equations with constant coefficients, General solution.

[1] Chapter 4 (Sections 4.1 to 4.5)

Weeks 7 and 8: The Cauchy problem for second order PDE, Homogeneous wave equation.

[1] Chapter 5 (Sections 5.1, 5.3, and 5.4)

Weeks 9 and 10: Initial boundary value problem, Non-homogeneous boundary conditions, Finite string with fixed ends, Non-homogeneous wave equation, Goursat problem.

[1] Chapter 5 (Sections 5.5 to 5.7, and 5.9)

Weeks 11 and 12: Method of separation of variables for second order PDE, Vibrating string problem.

[1] Chapter 7 (Sections 7.1 to 7.3)

Weeks 13 and 14: Existence (omit proof) and uniqueness of vibrating string problem. Heat conduction problem. Existence (omit proof) and uniqueness of the solution of heat conduction problem. Non-homogeneous problem.

[1] Chapter 7 (Sections 7.4 to 7.6, and 7.8)

MAT 609C : Riemann Integration

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: To understand the integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration. The sequence and series of real valued functions, and an important class of series of functions (i.e., power series).

Course Learning Outcomes: The course will enable the students to learn about:

- i) Some of the families and properties of Riemann integrable functions, and the applications of the fundamental theorems of integration.
- ii) Beta and Gamma functions and their properties.
- iii) The valid situations for the inter-changeability of differentiability and integrability with infinite sum, and approximation of transcendental functions in terms of power series.

Unit 1: Riemann Integration

(35 Marks, 5 Weeks)

Definition of Riemann integration, (Algebraic and order properties of Riemann Integrals) Boundedness theorem, Riemann integrability, Cauchy's criterion, Squeeze Theorem, Riemann integrability of step, continuous, and monotone functions, Additivity theorem, Fundamental theorems (First and Second forms), substitution theorem, Lebesgue's integrability criteria, composition theorem, product theorem, Integration by parts, Darboux sums, Darboux integrals, Darboux integrability criteria, equivalence of Riemann integral and Darboux integral.

Unit 2: Sequence and Series of Functions

(35 Marks, 5 Weeks)

Pointwise and uniform convergence of sequence of functions, Theorem on the continuity of the limit function of a sequence of functions, Theorems on the interchange of the limit and derivative, and the interchange of the limit and integrability of a sequence of functions. Pointwise and uniform convergence of series of functions, Theorems on the continuity, Derivability and integrability of the sum function of a series of functions, Cauchy criterion and the Weierstrass M-Test for uniform convergence.

Unit 3: Improper Integral and Power Series

(30 Marks, 4 weeks)

Improper integrals of Type-I, Type-II and mixed type, Convergence of Beta and Gamma functions, and their properties.

Definition of a power series, Radius of convergence, Absolute convergence (Cauchy-Hadamard theorem), Uniform convergence, Differentiation and integration of power series, Abel's Theorem.

References:

1. **Bartle, Robert G., & Sherbert, Donald R.** (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition. Delhi.
2. **Denlinger, Charles G.** (2011). *Elements of Real Analysis*. Jones and Bartlett (Student Edition). First Indian Edition. Reprinted 2015.

3. **Ghorpade, Sudhir R. & Limaye, B. V.** (2006). *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE). First Indian reprint.
4. **Ross, Kenneth A.** (2013). *Elementary Analysis: The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer.

Teaching Plan (MAT 609C : Riemann Integration):

Week 1: Definition of Riemann integration.

[1] Chapter 7 [Section (7.1.1 to 7.1.4)]

Week 2: Some properties of Riemann integral, Boundedness theorem,

[1] Chapter 7 [Section (7.1.5 to 7.1.7), Exercises of section 7 (1, 2, 7, 8)]

Week 3: Riemann integrable function, Cauchy criterion, Squeeze theorem, Riemann integrability of step, continuous, and monotone functions, additive theorem

[1] Chapter 7 [Section (7.2.1 to 7.2.13)]

Week 4: Fundamental theorems (First and Second forms), substitution theorem, Lebesgue's integrability criteria, product theorem, Integration by parts

[1] Chapter 7 [Section (7.3.1 to 7.3.17)]

Week 5: Darboux sums, Darboux integrals, Darboux integrability criteria, equivalence of Riemann integral and Darboux integral.

[1] Chapter 7 [Section (7.4.1 to 7.4.11)]

Week 6: Definitions and examples of pointwise and uniformly convergent sequence of functions.

[1] Chapter 8 [Section 8.1 (8.1.1 to 8.1.10)]

Week 7: Motivation for uniform convergence by giving examples. Theorem on the continuity of the limit function of a sequence of functions.

[1] Chapter 8 [Section 8.2 (8.2.1 to 8.2.2)]

Week 8: The statement of the theorem on the interchange of the limit function and derivative, and its illustration with the help of examples. The interchange of the limit function and integrability of a sequence of functions.

[1] Chapter 8 [Section 8.2 (Theorems 8.2.3, and 8.2.4)]

Week 9: Pointwise and uniform convergence of series of functions, Theorems on the continuity, derivability and integrability of the sum function of a series of functions.

[1] Chapter 9 [Section 9.4 (9.4.1 to 9.4.4)]

Week 10: Cauchy criterion for the uniform convergence of series of functions, and the Weierstrass M-Test for uniform convergence.

[2] Chapter 9 [Section 9.4 (9.4.5 to 9.4.6)]

Week 11: Improper integrals of Type-I, Type-II and mixed type.

[2] Chapter 7 [Section 7.8 (7.8.1 to 7.8.18)]

Week 12: Convergence of Beta and Gamma functions, and their properties.

[3] Pages 405 - 608C

Week 13: Definition of a power series, Radius of convergence, Absolute and uniform convergence of a power series.

[4] Chapter 4 (Section 23)

Week 14: Differentiation and integration of power series, Statement of Abel's Theorem and its illustration with the help of examples.

[4] Chapter 4 [Section 26 (26.1 to 26.6)]

MAT 610C : Numerical Analysis

Total Marks: 100 (Theory: 75 + Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 hrs) **Examination:** 3 Hrs.

Course Objectives: To comprehend various computational techniques to find approximate value for possible root(s) of non-algebraic equations, to find the approximate solutions of system of linear equations and ordinary differential equations. Also, the use of Computer Algebra System (CAS) by which the numerical problems can be solved both numerically and analytically, and to enhance the problem solving skills.

Course Learning Outcomes: The course will enable the students to learn the following:

- i) Some numerical methods to find the zeroes of nonlinear functions of a single variable and solution of a system of linear equations, up to a certain given level of precision.
- ii) Interpolation techniques to compute the values for a tabulated function at points not in the table.
- iii) Applications of numerical differentiation and integration to convert differential equations into difference equations for numerical solutions.

Unit 1: Methods for solving Algebraic and Transcendental Equations (30 Marks, 4 weeks)

Rate of Convergence, Methods of iteration, Bisection method, Newton-Raphson method, Fixed point iteration method, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel method.

Unit 2: Interpolation (35 Marks, 5 weeks)

Finite difference, relation between the operators, ordinary and divided differences, Newton's forward and Backward interpolation formulae, Newton's divided difference formulae and their properties, Lagrange, Hermite and Spline interpolation, Least square polynomial approximation.

Unit 3: Numerical Differentiation and Integration (35 Marks, 5 weeks)

First order and higher order approximation for first derivative, Approximation for second derivative. Numerical integration by Newton-Cotes formula, Trapezoidal rule, Simpson's rule and its error analysis. Methods to solve ODE's, Picard's method, Euler's and Euler's modified method and Runge-Kutta methods of 2nd and 4th order. Solution of boundary value problems of ordinary differential equations using Finite Difference method.

References:

1. **Bradie, Brian.** (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education, India. Dorling Kindersley (India) Pvt. Ltd. Third impression 2011.

Additional Readings:

1. **Jain, M. K., Iyengar, S. R. K., & Jain, R. K.** (2012). *Numerical Methods for Scientific and Engineering Computation*. (6th ed.). New Age International Publisher, India, 2016.
2. **Gerald, C. F., & Wheatley, P. O.** (2008). *Applied Numerical Analysis* (7th ed.). Pearson Education. India.
3. **Hildebrand F.B.** (2013). *Introduction to Numerical Analysis* (2nd Edn.) Dover publication.
4. **Schilling J. Robert, Harris I. Sandra** (1999). *Applied Numerical Methods for Engineering using MATLAB and C*. Books/Cole

Teaching Plan (MAT 610C : Numerical Analysis):

Week 1: Algorithms, Convergence, Order of convergence and examples. [1] Chapter 1 (Sections 1.1 and 1.2).

Week 2: Bisection method, False position method and their convergence analysis, Stopping condition and algorithms. [1] Chapter 2 (Sections 2.1 and 2.2).

Week 3: Fixed point iteration method, its order of convergence and stopping condition. [1] Chapter 2 (Section 2.3).

Week 4: Newton's method, Secant method, their order of convergence and convergence analysis. [1] Chapter 2 (Sections 2.4 and 2.5). Department of Mathematics, University of Delhi 49

Week 5: Examples to understand partial and scaled partial pivoting, LU decomposition. [1] Chapter 3 (Sections 3.2, and 3.5 up to Example 3.15).

Weeks 6 and 7: Application of LU decomposition to solve system of linear equations. Gauss–Jacobi method, Gauss–Seidel. [1] Chapter 3 (Sections 3.5 and 3.8).

Week 8: Lagrange interpolation: Linear and higher order interpolation, and error in it. [1] Chapter 5 (Section 5.1).

Weeks 9 and 10: Divided difference and Newton interpolation, Piecewise linear interpolation. [1] Chapter 5 (Sections 5.3 and 5.5).

Weeks 11 and 12: First and higher order approximation for first derivative and error in the approximation. Second order forward, Backward and central difference approximations for second derivative, Richardson extrapolation method [1] Chapter 6 (Sections 6.2 and 6.3).

Week 13: Numerical integration: Trapezoidal rule, Simpson's rule and its error analysis. [1] Chapter 6 (Section 6.4).

Week 14: Euler's method to solve ODE's, Second order Runge–Kutta methods: Modified Euler's method, Heun's method and optimal RK2 method. [1] Chapter 7 (Section 7.2 up to Page 562 and Section 7.4, Pages 582-585).

SEMESTER-V

MAT 711C : Metric Spaces

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs,

Course Objectives: The course aims at providing the basic knowledge pertaining to metric spaces such as open and closed balls, neighbourhood, interior, closure, subspace, continuity, compactness, connectedness etc.

Course Learning Outcomes: After completion of the course, a student will be able to

- i) understand the basic concepts of metric spaces and the concept such as open balls, closed balls
- ii) learn concepts of convergence of sequences, compactness, connectedness and their interrelations
- iii) correlate the concepts of Metric Space with the Analytical concepts such as Continuity and uniform continuity.

Unit 1: Basic Concepts

(30 marks, 4 weeks)

Metric spaces: Definition and examples, Open and closed ball, Neighbourhood, Open set, Interior, exterior, frontier and boundary points of a set, limit point of a set, derived set, closed set, closure of a set, diameter of a set, Dense set, Subspace of a metric space.

Unit 2: Complete Metric Spaces and Continuous Functions

(35 marks, 5 weeks)

Sequences in metric spaces, Cauchy and convergent sequences, Completeness of a metric space, Continuous mappings, Criteria for Continuity, Uniform Continuity, Homeomorphism, Lipschitz Conditions, Contraction mapping, Banach fixed point theorem.

Unit 3: Connectedness and Compactness

(35 marks, 5 weeks)

Connectedness, Components, Connected subsets of \mathbb{R} , Connectedness and continuity, Compactness, Compactness and Continuity, Sequential compactness, Compactness and finite intersection property, Bolzano-Weierstrass property, Heine-Borel theorem, Totally bounded sets, Compact Subsets of Function Spaces.

References:

1. E. T. Copson (1988). *Metric Spaces*. Cambridge University Press.
2. S. Kumaresan (2014). *Topology of Metric Spaces* (2nd ed.). Narosa Publishing House. New Delhi.
3. Satish Shirali & Harikishan L. Vasudeva (2006). *Metric Spaces*. Springer-Verlag.
4. Micheál O'Searcoid (2009). *Metric Spaces*. Springer-Verlag.

Additional reading:

1. P. K. Jain & Khalil Ahmad (2019). *Metric Spaces*. Narosa Publishing House. New Delhi.
2. G.F. Simmons (2004). *Introduction to Topology and Modern Analysis*. Tata McGraw Hill. New Delhi.

Teaching Plan (MAT 711C: Metric Spaces)

Week 1 : Metric spaces: Definition and examples[1] Chapter 2, [3] Chapter 1 section 1.2

Week 2 and 3: Open and closed ball, Neighbourhood, Open set, Interior, exterior, frontier and boundary points of a set limit point of a set, derived set, closed set, closure of a set, diameter of a set, Dense set, Subspace of a metric space. [1] Chapter 3

Week 4 and 5: Sequences in metric spaces, Cauchy and convergent sequences, Completeness of a metric space [1] Chapter 4, [2] Chapter 2

Week 6 and 7: Continuous mappings, Criteria for Continuity, Uniform Continuity, Homeomorphism [1] Chapter 7, [3] Chapter 3 section 3.1 – 3.5, [2] Chapter 3.

Week 8 : Lipschitz conditions, Contraction mapping, Banach fixed point theorem. [1] Chapter 8, [2] Chapter 6 section 6.4, [3] Chapter 3 section 3.7, [5] Chapter 9.

Week 9 and 10: Connectedness, Components, Connected subsets of \mathbb{R} , Connectedness and continuity [1] Chapter 5, [2] Chapter 5, [3] Chapter 4 section 4.1

Week 11, 12, 13 and 14: Compactness, Compactness and Continuity, Sequential compactness, Compactness and finite intersection property, Bolzano-Weierstrass property, Heine-Borel theorem, Totally bounded sets, Compact Subsets of Function Spaces. [1] Chapter 6, [2] Chapter 4, [4] Chapter 12 section 12.8.

MAT 712C : Mechanics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs,

Course Objectives: The course aims at providing the basic knowledge pertaining to dynamics such as simple harmonic motions, particle dynamics, projectiles, Kepler's Law, dynamics of rigid bodies. It also aims to study and impart knowledge on statics such as coplanar forces, catenary, stable and unstable equilibrium.

Course Learning Outcomes: This course will enable the students to:

- i) Deal with the kinematics and kinetics of the rectilinear and planar motions of a particle including the constrained oscillatory motions of particles.
- ii) Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions, which were deduced by him long before the mathematical theory given by Newton.
- iii) Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces acting on a rigid body.

Unit 1: Dynamics

(35 marks, 5 weeks)

Components of velocities and accelerations along, radial and transverse, along tangential and normal, Simple Harmonic motions, Dynamics of a particle, Motion on smooth and rough plane curves, Motion in resisting medium including projectile, Motion of varying mass, Central orbits and Kepler's Law, Acceleration in different Coordinate system

Unit 2: Statics

(35 marks, 5 weeks)

Equilibrium condition of coplanar forces, Equilibrium of strings, common catenary, catenary of uniform strength, Force in 3-dimension, Poinso's Central axis, Wrenches Null lines and planes, stable and unstable equilibrium

Unit 3: Dynamics of Rigid Bodies

(30marks, 4 weeks)

Moments and products of inertia, Momental Ellipsoid, Equipomental systems, Principal Axis, D'Alembert's Principle, Equations of motion of rigid bodies, Motion of centre of inertia, Motion relative to centre of inertia, Motion about a fixed axis, Compound Pendulum, Motion in 2 dimension under finite and impulsive forces, Conservation of momentum and Energy. Euler's dynamical equations for the motion of a rigid body about an axis, Theory of small oscillations.

References:

- 1 **S.L. Loney** (1988): *An elementary treatise on dynamics of particle and of rigid bodies*. Cambridge University Press 1956, reprinted by S. Chand & Company (P) Ltd.
- 2 **Das & Mukherjee** (2010): *Dynamics* published by S. Chand & company (p) Ltd. ISBN-81-85624-96-8.
- 3 **Das & Mukherjee** (2010): *Statics* published by S. Chand & company (p) Ltd., ISBN-81-85624-18-6.
- 4 **S.L. Loney** (2004): *An Elementary treatise on Statics* published by A.I.T.B.S., New Delhi, 2004 ISBN-81-7473-123-7.
- 5 **A.S. Ramsey** (2009): *Statics*, Cambridge University Press

Additional reading

- 1 **M. Ray and G.C. Sharma** (2008): *A Textbook of dynamics* published by S. Chand & company (p) Ltd., (Chapter 1,2,6,8,9,11,12), ISBN-81-219-0342-4.
- 2 **R.S. Verma**: *A Text Book on Statics*, Pothishala Pvt Ltd., Allahabad.
- 3 **A.S. Ramsey** (2009): *Dynamics*, Cambridge University Press
- 4 **P. L. Srivastava** (1964). *Elementary Dynamics*. Ram Narin Lal, Beni Prasad Publishers Allahabad.
- 5 **J. L. Synge & B. A. Griffith** (1949). *Principles of Mechanics*. McGraw-Hill

Teaching Plan (MAT 712C : Mechanics)

Week 1 : Components of velocities and accelerations along, radial and transverse, along tangential and normal
[1] Art 48. 49. 87, 88

Week 2: Simple Harmonic motions [1] Art 22-25, [2] Art 17.1 - 17.4. 17.6. 17.7

Week 3 : Dynamics of a particle, Motion on smooth and rough plane curves [1]Art 14.1, 14.2, 15.1, 15.2, 16.1, 16.2

Week 4: Motion in resisting medium including projectile, Motion of varying mass [1] Art 104-112

Week 5: Central orbit, Kepler's Law [1]Art 53-55, 57, 60, 64-67, 69-70), Acceleration in different Coordinate system [1] Art 125-127

Week 6 and 7: Equilibrium condition of coplanar forces [3]Art 81, 8.3, Equilibrium of strings, common catenary, catenary of uniform strength [3]Art 14.1-14.5 [5] Art 12.2, 12.21, 12.22, 12.5

Week 8 and 9 : Force in 3-dimension, Poinsots Central axis [1]Art 154-157, 162-165, [4] Art 184-186, 188-190

Week 10: Wrenches Null lines and planes [4]Art 206-208, stable and unstable equilibrium [4] Art 158, [1]Art 11.5, 11.6, 11.62, 11.7

Week 11 and 12: Moments and products of inertia [1]Art 144-149, Momental Ellipsoid [1]Art 151, Equipmomental systems, Principal Axis [1]Art 154, 155

Week 13: D'Alembert's Principle, Equations of motion of rigid bodies, Motion of centre of inertia, Motion relative to centre of inertia [1]Art 162 . Motion about a fixed axis [1]Art 168 -171, Compound Pendulum [1]Art 173-175

Week 14: Motion in 2 dimension under finite and impulsive forces [1]Art 187-190, Conservation of momentum and Energy. [1]Art 235, 236, 238, 239, 242, Euler's dynamical equations for the motion of a rigid body about an axis, Theory of small oscillations.

SEMESTER-VI

MAT 713C : Complex Analysis

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorials (per week)

Duration: 14 Weeks (70 Hrs. Theory) **Examination:** 3 Hrs.

Course Objectives: This course aims to introduce the basic ideas of analysis for complex functions in complex variables. Particular emphasis has been laid on Cauchy's theorems, series expansions and calculation of residues.

Course Learning Outcomes: The completion of the course will enable the students to:

- i) Understand the significance of differentiability of complex functions leading to the understanding of Cauchy-Riemann equations.
- ii) Evaluate the contour integrals and understand the role of Cauchy-Goursat theorem and the Cauchy integral formula.
- iii) Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues and apply Cauchy Residue theorem to evaluate integrals.

Unit 1: Analytic Functions and Cauchy-Riemann Equations (35 marks, 5 weeks)

Functions of complex variables, Limits, Theorems on limits, Limits involving the point at infinity, Continuity, Derivatives, Differentiability, Cauchy-Riemann equations, Sufficient conditions for differentiability; Polar forms of Cauchy-Riemann equations, Analytic functions, Trigonometric function, Multivalued Functions and its branches, Logarithmic functions, Complex exponents.

Unit 2: Complex Integrals (30 marks, 4 weeks)

Definite integrals of complex functions over a real interval, Contours, Contour integrals, Antiderivatives, Proof of antiderivative theorem, Cauchy-Goursat theorem, Cauchy integral formula; An extension of Cauchy integral formula, Extension of Cauchy integral formula, Cauchy's inequality, Liouville's theorem and the fundamental theorem of algebra.

Unit 3: Series and Residues (35 marks, 5 weeks)

Convergence of sequences and series, Taylor series and its examples; Laurent series and its examples, Absolute and uniform convergence of power series, Uniqueness of series representations, Singular points, Isolated singular points, Residues, Cauchy's residue theorem, residue at infinity; Types of isolated singular points, Residues at poles and its examples.

References:

1. **H.S. Kasana.** (2015). *Complex Variables, Theory and Applications*, (2nd ed.). Prentice Hall of India Learning Private Limited.
2. **Brown, James Ward, & Churchill, Ruel V.** (2014). *Complex Variables and Applications* (9th ed.). McGraw-Hill Education. New York

Additional Readings:

1. **Bak, Joseph & Newman, Donald J.** (2010). *Complex analysis* (3rd ed.). Undergraduate Texts in Mathematics, Springer. New York.
2. **Zills, Dennis G., & Shanahan, Patrick D.** (2003). *A First Course in Complex Analysis with Applications*. Jones & Bartlett Publishers, Inc.
3. **Mathews, John H., & Howell, Russell W.** (2012). *Complex Analysis for Mathematics and Engineering* (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.
4. **John B. Conway** (1973). *Functions of One Complex Variables*. Springer-Verlag.
5. **H.A. Priestley** (2003). *Introduction to Complex Analysis*, Oxford University Press.
6. **L.V. Ahlfors** (2017). *Complex Analysis* (3rd Edn.). McGraw Hill Education.

Teaching Plan (MAT 713C : Complex Analysis)

Week 1: Functions of a complex variable, limits, theorems on limits, limits involving the point at infinity, continuity. [1] Chapter 2 (2.1-2.3)

Week 2: differentiability, Cauchy- Riemann Equation, sufficient condition for differentiability, Polar form of Cauchy -Riemann Equation. [1] Chapter 2 (2.4-2.5)

Week 3: Analytic functions, Exponential, Trigonometric functions. [1] Chapter 2 (2.6), Chapter 3 (3.1, 3.2, 3.3)

Week 4: Multivalued Functions and its branches, Logarithmic functions, Complex Exponents. [1] Chapter 3 (3.5-3.7)

Week 5: Definite Integrals of complex functions over a real interval, Contours, Contours Integrals [2] Chapter 4 (42-46) , [1] Chapter 4 (4.1-4.2)

Week 6: Antiderivatives, Proof of antiderivative theorem, Cauchy-Goursat theorem, [2] Chapter 4 (48-53), [1] Chapter 4 (4.3-4.5)

Week 7: Cauchy Integral formula, Extension of Cauchy Integral formula, Consequences of Cauchy Integral formula. [2] Chapter 4 (54-57), [1] Chapter 4 (4.7)

Week 8 & 9: Chauchy's inequality, Liouville's theorem and fundamental theorem of algebra. [2] Chapter 4 (57-Theorem 3, 58), [1] Chapter 4 (4.8)

Week 10: Convergence of sequences and series, Taylor with examples, [2] Chapter 5 (60-65)

Week 11: Laurent series with examples, Absolute and uniform convergence of power series, uniqueness of series representation. [2] Chapter 5 (66- 69, 72)

Week 12 & 13: Singular points, Isolated singular points, Residues, Residue Theorem, Residue at infinity. [2] Chapter 6 (74-77)

Week 14: Types of isolated singular points, Residue at poles with examples. [2] Chapter 6 (78-81)

MAT 714C : Ring Theory & Linear Algebra

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of this course is to introduce the fundamental theory of two objects, namely - rings and vector spaces, and their corresponding homomorphisms. It aims to discuss the ring of polynomials and inner product spaces.

Course Learning Outcomes: On completion of this course, the students will be able to understand

- i) The fundamental concept of Rings, Fields, subrings, integral domains and the corresponding morphisms.
- ii) The concept of linear independence of vectors over a field, the idea of a finite dimensional vector space, basis of a vector space and the dimension of a vector space.
- iii) Basic concepts of linear transformations, the Rank-Nullity Theorem, matrix of a linear transformation, algebra of transformations, change of basis, eigen values and eigen vectors, orthogonality on vector spaces.

Unit 1: Rings :

(35 marks, 5 weeks)

Definition and examples of rings, properties of rings, Subrings, Integral Domains & Fields, Characteristics of a ring, Ideals, Ideal generated by a subset of ring, Factor rings, Operations on ideals, Prime Ideal, Principal Ideal and Maximal Ideals, Homomorphism and Isomorphism of Rings, Kernel of a homomorphism; First, Second and Third Isomorphism Theorems, Field of quotients, Polynomial ring over commutative ring, Division algorithm and consequences, Principal ideal domains, Reducibility and Irreducibility tests, Eisenstein's Irreducibility criteria, Unique factorisation in $\mathbb{Z}[x]$, Irreducibles, primes, Unique Factorization Domain(UFD), Euclidean domain,

Unit 2 : Vector Spaces :

(25 marks, 3 weeks)

Concept of vector space over a Field K , Subspaces, Necessary and sufficient condition for being a Subspace, Algebra of subspaces, Coset of subspace, Quotient Space, Linear combination of vectors, Linear Span, Subspace generated by a subset, Linear dependence & Linear independence, Basis and Dimension with related theorems, Finite Dimensional Vector Space, Dimension of Subspaces, Lagrange Interpolation formula.

Unit 3: Linear Transformations & Inner Product Space:

(40 marks, 6 weeks)

Linear Transformation, Null space, Ranges, Rank and Nullity of a Linear Transformation Kernel of Linear Transformation, Representation of Linear Transformation as matrices, Algebra of Linear Transformation, Isomorphism and Isomorphism theorem, Dual Space, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilator of a Subspace, Eigenvalues, Eigenvectors, Eigenspaces and Characteristic polynomial of a linear operator, Cayley-Hamilton theorem, The minimal polynomial for a linear operator.

Inner product spaces and Norms, Orthonormal basis, Gram-Schmidt Orthogonalization process Orthogonal Complements, Bessel's Inequality for Finite Dimensional Vector Spaces, S

References:

1. **Gallian, Joseph. A.** (2013). *Contemporary Abstract Algebra* (8th ed.). Cengage Learning India Private Limited. Delhi
2. **Friedberg, Stephen H, Insel, Arnold J, & Spence Lawrence E.** (2003). *Linear Algebra* (4th ed.) Prentice-Hall of India Pvt. Ltd. New Delhi.

Additional Readings:

1. **Herstein L.N.** (2006). *Topics in Algebra* (2nd ed.). Wiley Student Edition. India
2. **Hoffman, Kenneth, Kunze, Ray Alden** (1978). *Linear Algebra* (2nd ed.). Prentice-Hall of India Pvt. Ltd. New Delhi. Pearson Education India Reprint, 2015
3. **V.K. Khanna & S.K. Bhambri:** *A Course in Abstract Algebra*, Vikash Pub. House Pvt. Ltd, New Delhi.
4. **Gilbert Strang** (2014). *Linear Algebra and its Applications* (2nd Edn.), Elsevier.
5. **Vivek Sahai & Vikas Bist** (2013). *Linear Algebra* (3rd Edn.). Narosa Publishing House.
6. **Serge Lang** (2005). *Introduction to Linear Algebra* (2nd Edn.), Springer India.

Teaching Plan (MAT 714C : Ring Theory & Linear Algebra)

- Week 1:** Definition and examples of Ring, properties of Rings, subrings, Integral domain & Fields
[1] Chapter 12 & 13.
- Week 2&3:** Characteristic of a ring, ideals, ideals generated by a subset of a ring, Factor ring, Operation on ideals; Prime, Maximal & Principal ideals
[1] Chapter 13 & 14
- Week 4 :** Ring homomorphisms, Properties of ring homomorphisms; First, Second & Third Isomorphism theorems for rings, The field of Quotients. [1] Chapter 15 and Exercise 3 & 4 on page 346.
- Week 5:** Polynomial ring over commutative ring, Division algorithm and consequences, Principal ideal domains, Reducibility and Irreducibility tests, Einstein's Irreducibility criterions. Unique factorisation in $\mathbb{Z}[x]$, Irreducibles, primes, Unique Factorization Domain(UFD), Euclidean domain.
[1] Chapter 16 & 17,18
- Week 6:** Concept of vector space over a Field K, Subspaces, Necessary and sufficient condition for being a Subspace, Algebra of subspaces. [2] Chapter 1(Sections 1.2 and 1.3)
- Week 7:** Coset of subspace, Quotient Space, Linear combination of vectors, Linear Span, Subspace generated by a subset, Linear dependence & Linear independence.
[2] Chapter 1 (Section 1.4 and 1.5).
- Week 8:** Basis and Dimension with related theorems, Finite Dimensional Vector Space, Dimension of Subspaces, Lagrange Interpolation formula. [2] Chapter 1 (Section 1.6).
- Week 9:** Linear Transformation, Null space, Ranges, Rank and Nullity of a Linear Transformation Kernel of Linear Transformation. [2] Chapter 2(section 2.1)
- Week 10:** Representation of Linear Transformation as matrices, Algebra of Linear Transformation, Isomorphism and Isomorphism theorem. [2] Chapter 2 (section 2.2 , 2.3 & 2.4)
- Week 11:** Dual Space, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilator of a Subspace [2] Chapter 2 (section 2.6).
- Week 12:** Eigenvalues, Eigenvectors, Eigenspaces and Characteristic polynomial of a linear operator, Caley-Hamilton theorem, The minimal polynomial for a linear operator.
[2] Chapter 5(Section 5.1, 5.2 & 5.4), Chapter 7(Section 7.3, statement of theorem 7.16).
- Week 13 & 14:** Inner product spaces and Norms, Orthonormal basis, Gram-Schmidt Orthogonalization Process, Orthogonal Complements, Bessel's Inequality for Finite Dimensional Vector Spaces.
[2] Chapter 6(sections 6.1 & 6.2)

SEMESTER-VII

MAT 815C : Abstract Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week) 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course objectives: This course aims at studying topics in group theory, field theory and linear algebra at an advanced level. While studying this course, the student can develop his thinking and analytical skills.

Course Learning Outcomes:

After studying this course which comprises of three units, one each in group and ring theory, module theory and field theory, the student will be able to

- i). construct examples on composition series especially of finite groups along with applications and understand rings with chain conditions.
- ii). increase the thinking power of students in algebra more generally in module theory which is the generalization of vector spaces
- iii). identify field extensions such as algebraic extensions, transcendental extensions, separable extensions, normal extensions, splitting fields.

Preliminary: Coset and normal subgroups, homomorphism, simple groups, p-groups, symmetric groups, alternating groups, direct products, Sylow theorems, polynomial rings, fields, subfields, UFD, PID, ED.

Unit-1: Normal and Subnormal series, Composition Series, Zassenhaus Lemma, Schreier's refinement theorem, Jordan-Holder theorem, Commutator subgroup, Solvable Groups, Nilpotent groups, Noetherian and artinian rings, Hilbert basis theorem, (*refer chapter 6 and 19 of Bhattacharya et al, article 2.11, 3.8 of Vivek Sahai and Vikas Bist*)

Unit-2: Definition of modules with examples, module homomorphism and quotient modules, correspondence theorems, direct sum of modules, cyclic modules, simple modules. Semi-simple modules, Completely reducible modules, Schur's Lemma. Free modules, rings with invariance of rank property, direct sum and exact sequences, Noetherian and artinian modules (*refer chapter 14 of Bhattacharya et al chapter 4 of Vivek Sahai and Vikas Bist*)

Unit-3: Field extension, Algebraic and transcendental extensions, Separable and inseparable extensions, Normal extensions, Perfect fields, Finite fields, Primitive elements, Algebraically closed fields, (*refer chapter 15, 16 of Bhattacharya et al, chapter 5 of Vivek Sahai and Vikas Bist*)

References:

1. **P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul** (1997), *Basic Abstract Algebra* (2nd Edition), Cambridge University Press, Indian Edition.
2. **Vivek Sahai and Vikas Bist** (2008), *Algebra*, Narosa Publishing.

Additional Reading:

1. **I.N. Herstein** (2006), *Topics in Algebra*, Wiley Eastern Ltd., New Delhi.
2. **M. Artin** (1991), *Algebra*, Prentice-Hall of India.
3. **Dummit, David S. Foote, Richard M** (2016), *Abstract Algebra* (3rd Edn.) Wiley India
4. **P.M. Cohn** (1982, 1989, 1991), *Algebra*, Vols. I, II & III, John Wiley & Sons.

5. **N. Jacobson, W.H. Freeman** (1980), *Basic Algebra, Vols. I & II*, (also published by Hindustan Publishing Company).
6. **S. Lang** (1993), *Algebra*, 3rd edition, Addison-Wesley.
7. **I.S. Luther and I.B.S. Passi** (1996, 1999), *Algebra, Vol.I-Groups, Vol.II-Rings*, Narosa Publishing House.
8. **D.S. Malik, J.N. Mordeson, and M.K. Sen** (1997), *Fundamentals of Abstract Algebra*, McGraw-Hall, International Edition.

MAT 816C : Advanced Real Analysis

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week) 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The main objective of this course is to impart the various concepts of real numbers, sequences of functions, Riemann-Stieltjes Integration, functions in higher dimension.

Course Outcomes (COs)

At the end of the course, student will be able to understand

- i). The need to generalize the concept of integration to Riemann-Stieltjes Integration and will be able to apply its results.
- ii). Concepts of continuity and differentiability of functions in higher dimensional space and their application. They will have proper knowledge of Inverse function theorem and Implicit function theorem.
- iii). Concepts of measurable sets, non-measurable sets and integration with respect to a measure, difference between Lebesgue integration and Riemann integrations.

Preliminaries: Archimedean property, order completeness, Heine Borel Theorem, Cantor intersection theorem, Bolzano Weierstrass Theorem, Abel's and Dirichlet's tests for uniform convergence, Abel's theorem

Unit-1: Definition and existence of Riemann-Stieltjes integral, Properties of the Integral, Integration and differentiation, uniform convergence and Riemann-Stieltjes integration, the fundamental theorem of Calculus, Integration of vector-valued functions, Rectifiable curves.

Unit-2: Derivatives in an open subset of \mathbb{R}^n , Chain rules, Partial derivatives, interchange of the order of differentiation, Derivatives of higher orders, Taylor's theorem, Inverse function theorem, Implicit function theorem, Jacobians.

Unit-3: Lebesgue outer measure, Measurable sets, Regularity, Non measurable sets, Measurable functions, measurability of constant functions, measurability of continuous functions and measurability of sum, difference and product of measurable functions, Integration of Non negative function, Fatou Lemma, the general integral, integration of series, Riemann and Lebesgue integrals.

References

1. **G. De Barra** (1981), *Measure Theory and integration*, Wiley Eastern Ltd..
2. **Walter Rudin** (1976), *Principles of Mathematical Analysis* (3rd edition) McGraw-Hill, Kogakusha, International Student edition.

Additional Reading:

1. **T.M. Apostol** (1985), *Mathematical Analysis*, Narosa Publishing House, New Delhi.
2. **H.L. Royden** (1993), *Real Analysis*, Macmillan Pub. Co. Inc. 4th Edition, New York.
3. **A.J. White** (1968), *Real Analysis; An Introduction*, Addison- Wesley publishing Co., Inc.,.
4. **I.P. Natanson** (1961), *Theory of functions of a variable, Vol. 1*, Frederick Ungar Pub. Co..
5. **P.R. Halmos** (1950), *Measure Theory*, Van Nostrand, Princeton.
6. **P.K. Jain and V.P. Gupta** (2000), *Lebesgue Measure and Integration*, New Age International (P) Ltd., New Delhi.
7. **T.G. Hawkins** (1979), *Lebesgue's Theory of Integration: Its Origins and Development*, Chelsea, New York,.
8. **J.H. Williamson** (1962), *Lebesgue Integration*, Holt Rinehart and Winston, Inc., New York.

SEMESTER-VIII

MAT 817C : Topology

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week) 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course objectives: To understand clearly about the metric spaces, topological spaces, compactness of a general topological space so as to visualize many spaces with the concepts of countability.

Course Learning Outcomes

After completion of this course, the students may be able

- i). To have a clear concept of the fundamentals of Metric Space and Topological Space.
- ii). To check the countability of some spaces.
- iii). To analyze the distinction among the T_i -spaces.
- iv). To establish the interrelationship of the different types of compact spaces.
- v). To construct compactification of topological spaces.

Preliminaries: Equivalent sets, Countable and Uncountable Sets, Equivalence Relations, Axiom of choice, Zorn's lemma. Well-ordering theorem, Cantor Set, Inequalities: Triangle Inequality, Holder's Inequality, Cauchy-Schwarz Inequality, Minkowski's Inequality.

Unit – 1 : Metric and topological spaces: Definitions, Examples and Theorems – Open sets, Accumulation points, Derived Set, Closed sets, Closure. Dense subsets. Neighborhoods. Interior, exterior and boundary. Points, Convergent sequences, Coarser and finer topologies.

Metrics on Products, Incomplete and Complete Metric Spaces

Bases and sub-bases. Topology generated by classes of sets, Subspaces and relative topology.

Alternate methods of defining a topology in terms of Kuratowski Closure Operator and Neighbourhood Systems.

Unit – 2 : Continuous functions and homeomorphism (Metric space / topological space continued).

First and Second Countable spaces. Lindelof's theorems. Separable spaces. Second Countability and Separability.

Separation axioms T_0 , T_1 , $T_{3\frac{1}{2}}$, T_4 ; their Characterizations and basic properties. Urysohn's lemma. Tietze extension theorem.

Unit – 3 : Compactness. Continuous functions and compact sets. Basic properties of compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification. Compactness in metric spaces. Equivalence of compactness, countable compactness and sequential compactness in metric spaces.

Connectedness, Components, Locally connected spaces, Tychonoff product topology in terms of standard sub-base and its characterizations, Projection maps, Separation axioms and product spaces. Connectedness and product spaces, Compactness and product spaces (Tychonoff's theorem), Countability and product spaces.

References:

1. **James R. Munkres** (2002), *Topology*, Pearson Education Asia.

Additional Reading:

1. **J. Dugundji** (1966), *Topology*, Allyn and Bacon (Reprinted in India by Prentice Hall of India Pvt. Ltd.).
2. **George F. Simmons** (1963), *Introduction to Topology and Modern Analysis*, McGraw-Hall Book Company,.
3. **K.D. Joshi** (1983), *Introduction to General Topology*, Wiley Eastern Ltd.
4. **J.A. Kelley** (1995), *General Topology*, Van Nostrand, Reinhold Co., New York,.
5. **L. A. Steen and J.A. Seebach (Jr.)** (1970), *Counter examples in Topology*, Holt, Rinehart and Winston, New York.
6. **Crump W. Baker** (1991), *Introduction to Topology*, Wm C. Brown Publisher.
7. **M.J. Mansfield** (1963), *Introduction to Topology*, D. Van Nostrand Co. Inc. Princeton, N.J..
8. **B. Mendelson** (1962), *Introduction to Topology*, Allyn & Bacon Inc..
9. **B.K. Lahiri** (2000), *A First Course in Algebraic Topology*, Narosa.
10. **C.Wayne Patty** (2010), *Foundations of Topology*, Jones & Bartlet Student edition.
11. **P.K.Jain, Khalil Ahmad** (2019), *Metric Spaces*, Narosa, Third Edition.
12. **Michael O Searcold** (2008), *Metric Spaces*, Springer.

MAT 818C : Ordinary Differential Equations

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week) 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course objectives : To understand and analyze many physical phenomena described by ordinary differential equations as well as the theoretical aspects about the existence and uniqueness.

Course Learning Outcomes (s)

After studying the course the student will be able to understand

- i). The Picard theorem which guarantees a unique solution on some interval of the given differential equation, the Gronwall's inequality as an important tool to obtain various estimates in the theory of ordinary and stochastic differential equation which provides a comparison theorem that can be used to prove uniqueness of a solution to the initial value problem
- ii). The existence and asymptotic behavior of the Eigen-values, the corresponding qualitative theory of the Eigen functions and their completeness in the function space.
- iii). Analysis of the two-dimensional continuous dynamical system, the stability, stationary point and Lyapunov function etc.

Unit – 1: Initial value problem and Nagumo's and Osgood's criteria, Gronwall's inequality, Maximal and Minimal solutions, Upper and lower solution with example, Concepts of local existence, existence in the large and uniqueness of solutions with examples.

Unit – 2: Linear second order equations – Preliminaries, Basic facts, Self adjoint equation, Theorems of Sturm, Sturm-Liouville Boundary Value Problems, Number of zeros, Nonoscillatory equations and principal solutions, Nonoscillation theorems.

Second order Boundary value problems – Linear problems, Nonlinear problems, Green's function.

Unit – 3: Poincare-Bendixson Theory – Autonomous systems, Lyapunov function, Index of a stationary point, Poincare-Bendixson theorem, Stability of periodic solutions, rotation points, Foci, nodes and saddles points.

References :

1. **S.L. Ross** (1984), *Differential Equation*, Wiley Student Edition, India
2. **P Hartman** (1987), *Ordinary Differential Equations*, SIAM Second Edition

Additional reading :

1. **M.D. Raisinghania** (2013), *Advanced Differential Equation*; S. Chand, New Delhi
2. **C.H. Edwards & D. E. Penney** (2015), *Differential Equations and Boundary value Problem*, Fifth Edition, Pearson, New Delhi

Skill Enhancement Paper

SEMESTER-I

MAT 501S (a) : LaTeX

Total Marks: 100 (Theory: 37.5, Internal Assessment: 12.5 and Practical: 50)

Workload: 2 Lectures (per week), 2 Practicals of 2 hrs (per week per student)

Duration: 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

Course Objectives: The purpose of this course is to acquaint students with the latest typesetting skills, which shall enable them to prepare high quality typesetting, beamer presentation and webpages.

Course Learning Outcomes: After studying this course the student will be able to:

- i) Typeset mathematical formulas, use nested list, tabular & array environments.
- ii) Create or import graphics.
- iii) Use beamer to create presentation.

Unit 1: Getting Started with LaTeX

(15 marks, 4 weeks)

Introduction to TeX and LaTeX, Typesetting a simple document, Adding basic information to a document, Environments, Footnotes, Sectioning and displayed material.

Unit 2: Mathematical Typesetting with LaTeX

(20 marks, 6 weeks)

Accents and symbols, Mathematical Typesetting (Elementary and Advanced): Subscript/ Superscript, Fractions, Roots, Ellipsis, Mathematical Symbols, Arrays, Delimiters, Multiline formulas, Spacing and changing style in math mode.

Unit 3: Graphics and Beamer Presentation in LaTeX

(15 marks, 4 weeks)

Graphics in LaTeX, Simple pictures using PS Tricks, Plotting of functions, Beamer presentation.

References:

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Lamport, Leslie (1994). *LaTeX: A Document Preparation System*, User's Guide and Reference Manual (2nd ed.). Pearson Education. Indian Reprint.

Practical/Lab work to be performed in Computer Lab.

Practicals:

- [1] Chapter 9 (Exercises 4 to 10), Chapter 10 (Exercises 1 to 4 and 6 to 9),
Chapter 11 (Exercises 1, 3, 4, and 5), and Chapter 15 (Exercises 5, 6 and 8 to 11).

Teaching Plan (MAT 501S (a) : LaTeX):

Weeks 1 to 3: Introduction to TeX and LaTeX, Typesetting a simple document, Adding basic information to a document, Environments, Footnotes, Sectioning and displayed material.

[1] Chapter 9 (9.1 to 9.5)

[2] Chapter 2 (2.1 to 2.5)

Weeks 4 to 7: Accents of symbols, Mathematical typesetting (elementary and advanced): subscript/superscript, Fractions, Roots, Ellipsis, Mathematical symbols, Arrays, Delimiters, Multiline formulas, Spacing and changing style in math mode. [1] Chapter 9 (9.6 and 9.7) [2] Chapter 3 (3.1 to 3.3)

Weeks 8 to 10: Graphics in LaTeX, Simple pictures using PS Tricks, Plotting of functions.

[1] Chapter 9 (Section 9.8)

[1] Chapter 10 (10.1 to 10.3)

[2] Chapter 7 (7.1 and 7.2)

Weeks 11 to 14: Beamer presentation.

[1] Chapter 11 (Sections 11.1 to 11.4)

MAT 501S (b) : Computational Mathematics Laboratory

Total Marks: 100 (Theory: 37.5, Internal Assessment: 12.5 and Practical: 50)

Workload: 2 Lectures (per week), 2 Practicals of 2 hrs (per week per student)

Duration: 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

Course Objective: This course is designed to introduce the student to the basics of power point presentations and working with spread sheets. Also the students of mathematics will have the chance to gain essential skills involving computational mathematics software called Mathematica.

Course Learning Outcomes: On successful completion of the course, students will be able to

- i). Develop, manage power point presentations while preparing for presentations in seminars with additional skills such as inserting pictures, objects, multimedia etc.
- ii). Work out with excel files with skill of preparing charts to represent the information found in daily life situations.
- iii). Use mathematical software to plot the graph of various functions.

Unit-1: Power Point Presentation

(10 marks, 3 weeks)

Navigate the PowerPoint interface, creating new presentation from scratch – or by using beautiful templets, Add text, Pictures, Sound, Movies and Charts. Designing slides using themes, colours and special effects, Animate objects on slides, work with Master slides to make presentation easy.

Unit -2: Spreadsheets

(15 marks, 4 weeks)

Examine spreadsheet concepts and explore the Microsoft Office Excel environment, Create, Open and View a workbook. Save and print workbooks. Enter and Edit data. Modify a worksheet and workbook. Work with cell references. Learn to use functions and formulas. Create and edit charts and Graphics. Filter and sort table data. Work with pivot tables and charts. Import and Export data.

Unit -3: Mathematica

(25 marks, 7 weeks)

Getting Acquainted with the notation and convention, the Kernel and the Front End, Built- functions. Basic operations, Assignment and Replacement. Logical Relations, Sum and Products, Loops.

Two-Dimensional Graphics – plotting functions of a single variable, Additional Graphics Commands, Animations.

Three-Dimensional Graphics – plotting functions of two variables, Special three-dimensional plots.

Equation(s) solving commands, Matrix operations – vectors and matrices operations, eigenvalues and eigenvectors, trace, adjoint, inverse, diagonalization etc.

References:

1. **Binder, Donald & Erickson, Martin** (2011). *A student's guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. **Hillier and Hillier** (2003). *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheet*, Second Edition, McGraw-Hill.
3. **Eugene Don**, Ph. D., *Schaum's Outlines Mathematica*, Mc-Graw Hill (2009).

List of Practical to be performed at the Laboratory:

a) **PowerPoint Presentation:**

1. Change the fonts, colour of text on a slide
2. Add bullets or numbers to text
3. Format text as superscript or subscript
4. Insert a picture that is save on your local drive or an internal server
5. Insert a picture from the web
6. Insert shapes in your slide

b) **Spreadsheet:**

1. Format, enhance, and insert formulas in spreadsheet.
2. Move data within and between workbooks.
3. Maintain a workbook and create a chart in a spreadsheet.
4. Create, modify and manage a database table and query.
5. Create relationships between tables in a database.
6. Import and export data among word processing software, a spreadsheet and a database.
7. Merge data in a database with a word processing document.

c) **Mathematica:**

1. In an expression containing x, y, z replace all x, y, z by x^2 , y^2 and z^2 .
2. Find the sum of i) $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{100}$, ii) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ to ∞
3. Solve the equation i) $x^3 - x + 1 = 0$ for x, ii) Solve: $x - y = 1, x^2 - xy + y^2 = 10$
4. Plot the graph of $\sin x$ and $\cos x$ together, where $-\pi \leq x \leq \pi$
6. Plot the graph of the functions $\sin \pi x \sin \pi y$, where $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$

SEMESTER-II

MAT 502S (a) : Python Programming

Total Marks: 100 (Theory: 37.5, Internal Assessment: 12.5 and Practical: 50)

Workload: 2 Lectures (per week), 2 Practicals of 2 hrs (per week per student)

Duration: 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

Course Objective: This course is designed to introduce the student to the basics of programming using Python. The course covers the topics essential for developing well documented modular programs using different instructions and built-in data structures available in Python.

Course Learning Outcomes: On successful completion of the course, students will be able to

- i). Develop, document, and debug modular python programs to solve computational problems.
- ii). Select a suitable programming construct and data structure for a situation.
- iii). Use built-in strings, lists, sets, tuples and dictionary in applications.
- iv). Define classes and use them in applications.
- v). Use files for I/O operations.

Unit 1: Introduction to Programming using Python

(20 marks, 6 weeks)

Structure of a Python Program, Functions, Interpreter shell, Indentation. Identifiers and keywords, Literals, Strings, Basic operators (Arithmetic operator, Relational operator, Logical or Boolean operator, Assignment Operator, Bitwise operator). Building blocks of Python: Standard libraries in Python, notion of class, object and method.

Unit 2 : Creating Python Programs

(15 marks, 4 weeks)

Input and Output Statements, Control statements:-branching, looping, Exit function, break, continue and pass, mutable and immutable structures. Testing and debugging a program.

Unit 3 : Visualization using 2D and 3D graphics and data structures

(15 marks, 4 weeks)

Visualization using graphical objects like Point, Line, Histogram, Sine and Cosine Curve, 3D objects, Built-in data structures: Strings, lists, Sets, Tuples and Dictionary and associated operations. Basic searching and sorting methods using iteration and recursion.

References:

1. Downey, A.B., (2015), *Think Python–How to think like a Computer Scientist*, 3rd edition. O'Reilly Media.
2. Taneja, S. & Kumar, N., (2017), *Python Programming-A Modular Approach*. Pearson Education.

Additional Reading:

1. Brown, M. C. (2001). *The Complete Reference: Python*, McGraw Hill Education.
2. Dromey, R. G. (2006), *How to Solve it by Computer*, Pearson Education.
3. Guttag, J.V. (2016), *Introduction to computation and programming using Python*. MIT Press.
4. Liang, Y.D. (2013), *Introduction to programming using Python*. Pearson Education.

Practical

1. Execution of expressions involving arithmetic, relational, logical, and bitwise operators in the shell window of Python IDLE.
2. Write a Python function to produce the outputs such as:
 - a)

```
      *
    * * *
  * * * * *
    * * *
      *
```


(b)

1
232
34543
4567654
567898765

3. Write a Python program to illustrate the various functions of the “Math” module.
4. Write a function that takes the lengths of three sides: **side1**, **side2** and **side3** of the triangle as the input from the user using the **input** function and return the area of the triangle as the output. Also, assert that the sum of the length of any two sides is greater than the third side.
5. Consider a showroom of electronic products, where there are various salesmen. Each salesman is given a commission of 5%, depending on the sales made per month. In case the sale done is less than 50000, then the salesman is not given any commission. Write a function to calculate total sales of a salesman in a month, commission and remarks for the salesman. Sales done by each salesman per week are to be provided as input. Assign remarks according to the following criteria:
 Excellent: Sales ≥ 80000
 Good: Sales ≥ 60000 and < 80000
 Average: Sales ≥ 40000 and < 60000
 Work Hard: Sales < 40000
6. Write a Python function that takes a number as an input from the user and computes its factorial.
7. Write a Python function to return nth terms of the Fibonacci sequence
8. Write a function that takes a number with two or more digits as an input and finds its reverse and computes the sum of its digits.
9. Write a function that takes two numbers as input parameters and returns their least common multiple and highest common factor.
10. Write a function that takes a number as an input and determines whether it is prime or not.
11. Write a function that finds the sum of the terms of the following series:
 a) $1 - x^2/2! + x^4/4! - x^6/6! + \dots x^n/n!$
 b) $1 + x^2/2! + x^4/4! + x^6/6! + \dots x^n/n!$
12. Write a Python function that takes two strings as an input from the user and counts the number of MATHing characters in the given pair of strings.
13. Write a Python function that takes a string as an input from the user and displays its reverse.
14. Write a Python function that takes a string as an input from the user and determines whether it is palindrome or not.
15. Write a Python function to calculate the sum and product of two compatible matrices
16. Write a function that takes a list of numbers as input from the user and produces the corresponding cumulative list where each element in the list present at index i is the sum of elements at index $j \leq i$.
17. Write a function that takes **n** as an input and creates a list of n lists such that i^{th} list contains first five multiples of i.
18. Write a function that takes a sentence as input from the user and calculates the frequency of each letter. Use a variable of dictionary type to maintain the count.
19. Write a Python function that takes a dictionary of **word:meaning** pairs as an input from the user and creates an inverted dictionary of the form meaning: list-of-words.
20. Usage of Python debugger tool-pydb and Python Tutor.
21. Implementation of Linear and binary search techniques
22. Implementation of selection sort, insertion sort, and bubble sort techniques

23. Write a menu-driven program to create mathematical 3D objects Curve, Sphere, Cone, Arrow, Ring, and Cylinder.
24. Write a program that makes use of a function to accept a list of n integers and displays a histogram.
25. Write a program that makes use of a function to display sine, cosine, polynomial and exponential curves.
26. Write a program that makes use of a function to plot a graph of people with pulse rate p vs. height h. The values of p and h are to be entered by the user.
27. Write a function that reads a file **file1** and displays the number of words and the number of vowels in the file.
28. Write a Python function that copies the content of one file to another.
29. Write a function that reads a file **file1** and copies only alternative lines to another file **file2**. Alternative lines copied should be the odd numbered lines.

Teaching Plan (MAT 502S (a) : Python Programming)

Week 1: Python Programming: An Introduction Structure of a Python program, understanding Python interpreter/Python shell, indentation. Atoms, identifiers and keywords, literals, Python strings, arithmetic operator, relational operator, logical or Boolean operator, bit wise operators.

Week 2: Variables and Functions Python standard libraries such as sys and math. Variables and assignment statements. Built-in functions such as input and print.

Week 3-4: Control Structures if conditional statement and for loop, While loop, break, continue, and pass statement, else statement. Infinite loop

Week 5: Functions Function definition and call, default parameter values, keyword arguments, assert statement

Week 6: Strings and Lists Strings-slicing, membership, and built-in functions on strings Lists- list operations.

Week 7: Mutable object Lists- built-in functions, list comprehension, passing list as arguments, copying list objects.

Week 8: Sets, tuples, and dictionary- associated operations and built-in functions.

Week 9: Testing and Debugging Determining test cases, use of python debugger tool- pydb for debugging

Week 10: Searching and Sorting Linear search, binary search, selection sort, insertion sort, and bubble sort

Week 11: Python 2D and 3D Graphics Visualization using graphical objects like point, line, histogram, sine and cosine curve, 3D objects

Week 12: File Handling Reading and writing text and structured files.

Week 13: Errors and Exceptions Types of errors and exceptions, and exception handling

Week 14: Classes Notion of class, object, and method.

MAT 502S (b) : Computer Algebra Systems and Related Software

Total Marks: 100 (Theory: 37.5, Internal Assessment: 12.5 and Practical: 50)

Workload: 2 Lectures (per week), 2 Practicals of 2 hrs (per week per student)

Duration: 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

Course Objectives: This course aims at familiarizing students with the usage of computer algebra systems (/Mathematica/MATLAB/Maxima/Maple) and the statistical software **R**. The basic emphasis is on plotting and working with matrices using CAS. Data entry and summary commands will be studied in **R**. Graphical representation of data shall also be explored.

Course Learning Outcomes: This course will enable the students to:

- i) Use CAS as a calculator, for plotting functions, animations and various applications of matrices.
- ii) Understand the use of the software **R** for entry, summary calculation, pictorial representation of data and exploring relationship between data.
- iii) Analyze, test, and interpret technical arguments on the basis of geometry.

Unit 1: Introduction to CAS and Applications

(15 marks, 4 weeks)

Computer Algebra System (CAS), Use of a CAS as a calculator, Computing and plotting functions in 2D, Plotting functions of two variables using Plot3D and Contour Plot, Plotting parametric curves surfaces, Customizing plots, Animating plots, Producing tables of values, working with piecewise-defined functions, Combining graphics.

Unit 2: Working with Matrices

(15 marks, 4 weeks)

Simple programming in a CAS, Working with matrices, Performing Gauss elimination, operations (transpose, determinant, inverse), Minors and cofactors, Working with large matrices, Solving system of linear equations, Rank and nullity of a matrix, Eigenvalue, eigenvector and diagonalization.

Unit 3: R - The Statistical Programming Language

(20 marks, 6 weeks)

R as a calculator, Explore data and relationships in **R**. Reading and getting data into **R**: Combine and scan commands, Types and structure of data items with their properties. Manipulating vectors, Data frames, Matrices and lists. Viewing objects within objects. Constructing data objects and conversions, Summary commands: Summary statistics for vectors, Data frames, Matrices and lists. Summary tables. Stem and leaf plot, Histograms. Plotting in **R**: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts and bar charts. Copy and save graphics to other applications.

References:

1. **Bindner, Donald & Erickson, Martin.** (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. **Torrence, Bruce F., & Torrence, Eve A.** (2009). *The Student's Introduction to Mathematica®: A Handbook for Precalculus, Calculus, and Linear Algebra* (2nd ed.). Cambridge University Press.
3. **Gardener, M.** (2012). *Beginning R: The Statistical Programming Language*, Wiley.

Note: Theoretical and Practical demonstration should be carried out only in **one** of the CAS: Mathematica/MATLAB/Maxima/Scilab or any other.

Practical/Lab work to be performed in Computer Lab.

Practicals:

- [1] Chapter 12 (Exercises 1 to 4 and 8 to 12), Chapter 14 (Exercises 1 to 3)
[2] Chapter 3 [Exercises 3.2 (1 and 2), 3.3 (1, 2 and 4), 3.4 (1 and 2), 3.5 (1 to 4), 3.6 (2 and 3)].
[2] Chapter 6 (Exercises 6.2 and 6.3).
[2] Chapter 7 [Exercises 7.1 (1), 7.2, 7.3 (2), 7.4 (1) and 7.6].

Note: Relevant exercises of [3] Chapters 2 to 5 and 7 (The practical may be done on the database to be downloaded from <http://data.gov.in/>).

Teaching Plan (Theory of MAT 502S (b): Computer Algebra Systems and Related Software):

Weeks 1 to 3: Computer Algebra System (CAS), Use of a CAS as a calculator, Computing and plotting functions in 2D, Producing tables of values, Working with piecewise-defined functions, Combining graphics. Simple programming in a CAS. [1] Chapter 12 (Sections 12.1 to 12.5). [2] Chapter 1, and Chapter 3 (Sections 3.1 to 3.6 and 3.8).

Weeks 4 and 5: Plotting functions of two variables using Plot3D and contour plot, Plotting parametric curves surfaces, Customizing plots, Animating plots. [2] Chapter 6 (Sections 6.2 and 6.3). **Weeks 6 to 8:** Working with matrices, Performing Gauss elimination, Operations (Transpose, Determinant, Inverse), Minors and cofactors, Working with large matrices, Solving system of linear equations, Rank and nullity of a matrix, Eigenvalue, Eigenvector and diagonalization. [2] Chapter 7 (Sections 7.1 to 7.8).

Weeks 9 to 11: R as a calculator, Explore data and relationships in R. Reading and getting data into R: Combine and scan commands, Types and structure of data items with their properties. Manipulating vectors, Data frames, Matrices and lists. Viewing objects within objects. Constructing data objects and conversions. [1] Chapter 14 (Sections 14.1 to 14.4). [3] Chapter 2, and Chapter 3. **Weeks 12 to 14:** Summary commands: Summary statistics for vectors, Data frames, Matrices and lists. Summary tables. Stem and leaf plot, histograms. Plotting in R: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts and Bar charts. Copy and save graphics to

Generic Elective (GE) Course - Mathematics

SEMESTER-III

MAT 601G : Quantitative Aptitude

Total Marks: 100 (Theory: 70, Internal Assessment: 30)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination: 3 Hrs.**

Course Objectives: The main aim of this course is to gain knowledge of elementary ideas about arithmetic abilities which one finds in daily life. It will help the students from any background to get acquainted with this knowledge and get prepared for any competitive examinations.

Course Learning Outcomes: This course will enable the students to:

- i) gain sufficient ideas of mental and arithmetic abilities.
- ii) handle mental/quantitative aptitude test questions with great ease.
- iii) acquire the skill of solving problems of daily life quickly.

Unit-1: Arithmetic Ability I

(30 marks, 4 weeks)

Chain Rule – Time and Work – Pipes and Cisterns

Time and Distance – Problems on Trains – Boats and Streams

Unit-2: Arithmetic Ability II

(30 marks, 4 weeks)

Simple Interest – Compound Interest – Stocks and Shares. (*Chapters 17, 18 & 19*)

Clocks – Area (*Chapters 24, 25*)

Unit-3: Arithmetic Ability III

(40 marks, 6 weeks)

Volume and Surface Area. (*Chapters 28*)

Permutations and Combinations. (*Chapters 30 & 31*)

Text Book:

1. Scope and treatment as in “*Quantitative Aptitude*”, S. Chand and Company Ltd. Ram Nagar, New Delhi (2007).

Teaching plan (MM 601G: Quantitative Aptitude):

Week 1 & 2: Chain Rule – Time and Work – Pipes and Cisterns, [1] Chapters 14, 15 & 16.

Week 3 & 4 : Time and Distance – Problems on Trains – Boats and Streams [1] Chapters 21, 22 & 29.

Week 5 & 6: Simple Interest – Compound Interest – Stocks and Shares. [1] Chapters 17, 18 & 19.

Week 7 & 8: Clocks – Area [1] Chapters 24, 25.

Week 9: Volume and Surface Area. [1] Chapter 28.

Week 10 to 14: Permutations and Combinations. [1] Chapters 30 & 31.

SEMESTER-IV

MAT 602G : Basic Tools of Mathematics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of the course is to introduce the concept of geometry, vectors algebraic ideas like various forms of mean, progression, polynomial will be taught to the students. The concept of differential calculus and probability will help the students in understanding their respective core courses with great comfort.

Course learning Outcomes: After studying this course, the student may understand

1. The basic concepts of Geometry and Vectors Analysis.
2. Some topics of Algebra and Differential Calculus.
3. Application of partial differentiation in daily life problems.
4. Properties and methods of Integration, solving of definite and indefinite integrals.
5. Basic ideas of probability such as probability distribution, expectations, Binomial Distribution, Poisson distribution, etc.

UNIT-1: Geometry and Vectors:

(40 marks, 6 weeks)

Geometry

Three-dimensional space, Rectangular Cartesian Coordinates, Polar Coordinates, Cylindrical Coordinates, Spherical coordinates. Change of origin, Section of a line joining two given points.

Vectors

Addition of two or more vectors, Negative of a vector, Subtraction of two vectors, Multiplication of a vector by a scalar, Vector equations, Collinear vectors, Position vector of a point, Section Ratio of a point, Linear combination of a set of vectors, Coordinates of two and three-dimensional vectors. Product of two or three vectors.

UNIT-2: Algebra and Calculus

(40 marks, 6 weeks)

Algebra

Geometric Mean, Arithmetic Mean, Harmonic Mean and related Inequalities, Arithmetic and Geometric Progression, Polynomial, Equation, Linear Equation, Quadratic Equation, Roots and Coefficients, Fundamental Theorem of Algebra, Binomial Theorem, Permutation, and Combination, Mathematical Induction, Determinants, Matrices, Solution of equations by matrix method.

Differential Calculus

Mappings, Inverse Mapping and Composite Mappings.

Limit, Continuity, Differentiation, Maxima and Minima, Tangent and normal, Partial Differentiation.

Integral Calculus

Definition, Properties, Methods of Integration, Definite integrals, Infinite Integrals.

UNIT-3: Probability

(20 marks, 2 weeks)

Probability

Definition, Random variable (discrete and continuous), Probability Distribution (mass function, density function, distribution function), Expectations, Some Standard Probability Distributions (Distributions: Binomial, Poisson, Negative Binomial, Geometric, Hyper-geometric, Normal, Exponent, Uniform, Gamma, Beta, etc.)

Recommended books

1. **B.S. Vatssa** : *Discrete Mathematics* ch.1, 2e, Wishwa Prakashan (A Division of Wiley Eastern Ltd.)
2. **Chandrika Prasad**: *Algebra and Theory of Equations*, Pothisala Pvt. Ltd.
3. **Das and Mukherjee**: *Differential Calculus*, UN Dhur & Sons Pvt. Ltd.
4. **Das and Mukherjee**: *Integral Calculus*, UN Dhur & Sons Pvt. Ltd.
5. **Ghosh & Maity**: *Vector Analysis*, New Central Book Agency, Kolkata
6. **S.C. Gupta and V.K. Kapoor**: *Fundamentals of Mathematical Statistics*, Sultan Chand & Sons.
7. **Chakraborty & Ghosh**: *Analytical Geometry and Vector Analysis*, UN. Dhur & Sons, Kolkata
8. **Chakraborty & Ghosh**: *Advanced Analytical Geometry*, UN. Dhur & Sons, Kolkata

SEMESTER - V

MAT 703G : Recreational Mathematics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The main objective of this course is to impart the knowledge of Mathematics used in daily life to the students with little background in the subject and to provoke enthusiasm.

Course Learning Outcomes(s)

After studying this course, the students will be able

- i). To understand basic set theory, mathematical puzzles, beauty of figurate numbers and to solve real-life problems.
- ii). To understand CRT, Fermat's Little Theorem, Euler's Theorem, Wilson's Theorem, application of congruences, application of Mathematics in Nature, Geometric shapes, patterns, etc.
- iii). To understand the application of Number Theory in ISSN, ISBN, UPC, Credit card check and have a knowledge about some mathematicians viz, Ramanujan, Hardy, Erdos etc.

Unit-1: Basic Set Theory and Fundamentals

(30 marks, 4 weeks)

Notations, Venn Diagram, Union, Intersection, Complement, Comparable, sets of Numbers, Line Diagram of the Number System, Intervals, Algebra of Sets.

Order Relation, Absolute Value, Summation Notation, Indexed Summation, Product Notation, Well Ordering Principle.

Recursion, Handshake Problem, Tower of Brahma, Binomial Theorem, Pascal's Identity, Pascal's Triangle, Magic Squares, Geometrical Patterns.

Polygonal, Triangular, Square, Pentagonal, Hexagonal, Pyramidal. Triangular Pyramidal, Square Pyramidal, Pentagonal Pyramidal, Hexagonal Pyramidal numbers.

Unit 2: Congruences

(35 marks, 5 weeks)

Basic properties of congruences, congruence classes, linear congruence, solutions, Chinese Remainder Theorem, Some special theorems, Fermat's little theorem, Euler's theorem, Wilson's theorem, Application of congruence; Divisibility test, check digits.

Detection of errors in an ISBN, ISSN, product code(UPC), credit card check digit, application of congruences in sports, setting time table for tournaments.

Unit 3: Some Applications and Biography of some Mathematicians

(35 marks, 5 weeks)

Palindromic number, Taxicab number (Hardy-Ramanujan Number).

Pythagorean Triples, Pythagorean triples and the unit circle.

Fibonacci Numbers, Fibonacci sequence, Fibonacci Problem, Dynamical Growth of rabbit population and Fibonacci sequence, Some Fascinating Numbers of Lucas, Examples of Mathematics in Nature, Geometric shapes, Symmetry, Fibonacci Spiral, Golden Ratio, Fractals.

Historical Notes on S. Ramanujan, G. Hardy, Paul Erdos, Aryabhata, Brahmagupta, Bhaskara.

References:

1. **Seymour Lipschutz**, *Set Theory and Related Topics*, *Schaum's Outline Series*, TMH/McGraw Hill
2. **Thomas Koshy** (2007): *Elementary Number Theory with Applications*, (Second Edn.), Elsevier.
3. **Joseph H. Silverman** (2014): *A Friendly Introduction to Number Theory*, (4th Edn.), Pearson IN.
4. **Neville Robbins**: *Beginning Number Theory*, (2nd Edn.), Jones & Barlett Learning.
5. **M.K. Sen, B.C. Chakraborty** (2002): *Introduction to Discrete Mathematics*, NCBA Publishers.
6. **Wikipedia**

Teaching Plan (MAT 703G : Recreational Mathematics)

Week 1: Venn Diagram, Union, Intersection, Complement, Comparable, sets of Numbers, Line Diagram of the Number System, Intervals, Algebra of Sets. ([1] Ch.1,2, 3, 7)

Week 2: Order Relation, Absolute Value, Summation Notation, Indexed Summation, Product Notation, Well Ordering Principle ([2] Ch. 1 sec 1.1 to 1.3)

Week 3: Recursion, Handshake Problem, Tower of Brahma, Binomial Theorem, Pascal's Identity, Pascal's Triangle, Magic Squares, Geometrical Patterns ([2] Ch. 1 sec 1.4, 1.5).

Week 4: Polygonal, Triangular, Square, Pentagonal, Hexagonal, Pyramidal. Triangular Pyramidal, Square Pyramidal, Pentagonal Pyramidal, Hexagonal Pyramidal ([2] Ch.1 Sec 1.6, 1.7)

Week 5: Congruences, congruence classes, linear congruence, solutions, Chinese Remainder Theorem ([2] Ch. 4, [4] Ch. 4 Sec. 4.2, 4.3)

Week 6 and 7 : Chinese Remainder Theorem, Some special theorems, Fermat's little theorem, Euler's theorem, Wilson's theorem. ([2] Ch. 6, 7, [5] Ch. 3)

Week 8 and 9: Application of congruence; Divisibility test, check digits, ISBN, ISSN, product code(UPC), credit card check digit, application of congruences in sports, setting timetable for tournaments ([2] Ch.5 sec 5.1, 5.3,5.5 , [5] Ch.4)

Week 10: Palindromic number, Taxicab number (Hardy-Ramanujan Number) [6], Pythagorean Triples, Pythagorean triples and the unit circle ([3] Ch. 2 & 3), [6].

Week 11, 12 and 13: Fibonacci Numbers, Fibonacci sequence, Fibonacci Problem, Dynamical Growth of rabbit population and Fibonacci sequence, Some Fascinating Numbers of Lucas, Examples of Mathematics in Nature, Geometric shapes, Symmetry, Fibonacci Spiral, Golden Ratio, Fractals ([2] Ch.2 Sec 2.6, ..) [6]

Week 14: Historical Notes on S. Ramanujan, G. Hardy, Paul Erdos, Aryabhata, Brahmagupta, Bhaskara [6].

SEMESTER - VI

MAT 704G : Discrete Mathematics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The course introduces formal logic notation, methods of proof, mathematical induction, set theory, permutations and combinations and counting principles. One can learn the concepts of lattices and Boolean algebra in analysis of various applications.

Course Learning Outcomes: This course will enable the students to:

- i) Understand the basic principles of logic, set theory, lattices and Boolean algebra.
- ii) Understand the ideas of basic counting techniques.
- iii) Proficiently construct logical arguments and rigorous proofs.

Unit 1: Logical Mathematics

(30 marks, 4 weeks)

Compound statements (and, or, implication, negation, contrapositive, quantifiers), Proofs in Mathematics, Truth tables, Basic logical equivalences and its consequences, Logical arguments, Binary relations, Types of binary relations, Equivalence relations, Partial and total ordering (Hasse diagram, Lexicographic order, Isomorphism, extremal elements).

Unit 2: Lattices and its Properties

(35 marks, 5 weeks)

Lattices, Duality principle, Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices, Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn–McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.

Unit 3: Applications of Numbers

(35 marks, 5 weeks)

Properties of integers, Division algorithm, Divisibility and Euclidean algorithm, GCD, LCM, Relatively prime, Prime numbers, Statement of fundamental theorem of arithmetic, Fermat primes, Recursively defined sequences, Recursive relations and its solution (characteristics polynomial and generating function), Principles of counting (Inclusion/Exclusion, Addition and Multiplication rule, Pigeon-Hole).

References:

1. **Bernard Kolman, Robert C Busby, Sharon C Ross** (2004). *Discrete Mathematical Structures*. (Fifth Edition) Pearson Education, Inc..
2. **Goodaire, Edgar G., & Parmenter, Michael M.** (2005). *Discrete Mathematics with Graph Theory* (3rd ed.). Pearson Education (Singapore) Pvt. Ltd. Indian Reprint.
3. **Lidl, Rudolf & Pilz, Günter.** (1998). *Applied Abstract Algebra* (2nd ed.). Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint 2004.

Additional Reading:

1. **Rosen, Kenneth H.** (2012) *Discrete Mathematics and its Applications* (7th ed.). McGraw-Hill Education (India) Pvt. Ltd.

Teaching Plan (MAT 704G : Discrete Mathematics)

Week 1: Compound statements (and, or, implication, negation, contrapositive, quantifiers), Proofs in Mathematics [2] Chapter 1 (section 1.1, 1.2)

Week 2: Truth tables, Basic logical equivalences and its consequences [2] Chapter 1 (section 1.3, 1.4)

Week 3: Logical arguments, Binary relations [2] Chapter 1(section 1.5) Chapter 2(section 2.3)

Week 4: Types of binary relations, Equivalence relations, Partial and total ordering, (Hasse diagram, Lexicographic order, Isomorphism, extremal elements)

[2] Chapter 2(section 2.4, 2.5), [1] Chapter 6(section 6.1, 6.2)

Week 5 & 6: Lattices, Duality principle, Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices

[1] Chapter 6 (section 6.3), [3] Chapter 1(section 1& 2)

Week 7 & 8: Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn–McCluskey method, Karnaugh diagrams,

[3] Chapter 1 (section 3, 4, 6) [1] Chapter 6 (section 6.4 , 6.5, 6.6)

Week 9: Switching circuits and applications of switching circuits. [3] Chapter 2(section 7 & 8)

Week 10: Properties of integers, Division algorithm, [2] Chapter 4(section 4.1 to 4.1.6)

Week 11: Divisibility and Euclidean algorithm, GCD, LCM, Relatively prime [2] Chapter 4(section 4.2)

Week 12: Prime numbers, Statement of the fundamental theorem of arithmetic, Fermat primes,

[2] Chapter 4(section 4.3)

Week 13: Recursive relations and its solution (characteristics polynomial and generating function),

[2] Chapter 5(section 5.2, 5.3, 5.4)

Week 14: Principles of counting (Inclusion/Exclusion, Addition and Multiplication rule, Pigeon-Hole)

[2] Chapter 6 (section 6.1, 6.2, 6.3)

SEMESTER-VII

MAT 805G : Analytical Geometry and Theory of Equations

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students with certain ideas about conic sections, vectors in coordinate system and general properties of roots of polynomial equations with some applications.

Course Learning Outcomes: After completion of this paper, the students will be able to:

- i). Classify and sketch conics four different types of conic sections – the circle, the ellipse, the hyperbola and the parabola – in Cartesian and polar coordinates.
- ii). Visualize three dimensional objects – spheres and cylinders – using vectors.
- iii). Understand the properties of roots of polynomial equations.

Unit 1: Conic Sections, Parametrized Curves, and Polar Coordinates

Conic sections and quadratic equations: Circle, Parabola, Ellipse, and hyperbola; Techniques for sketching: Parabola, Ellipse, and Hyperbola; Reflection properties of parabola, ellipse, and hyperbola, Classifying conic sections by eccentricity, Classification of quadratic equations representing lines, parabola, ellipse, and hyperbola; Parameterization of plane curves, Conic sections in polar coordinates and their sketching.

Unit 2: Three-Dimensional Space: Vectors

Rectangular coordinates in 3-space, Spheres and cylindrical surfaces, Vectors viewed geometrically, Vectors in coordinate systems, Vectors determined by length and angle, Dot product, Cross product and their geometrical properties, Parametric equations of lines in 2-space and 3-space.

Unit 3: Theory of Equations

General properties of polynomials and equations, Descartes' rule of signs for positive and negative roots, Relation between the roots and the coefficients of equations, Applications, Depression of an equation when a relation exists between two of its roots, Symmetric functions of the roots and its applications, Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Graphical representation of derived function, Rolle's theorem, Multiple roots of the equation.

References:

1. **Thomas, Jr. George B., Weir, Maurice D., & Hass, Joel** (2014). *Thomas' Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.
2. **Burnside, W.S., & Panton, A.W.** (1979), *The Theory of Equations*, Vol. 1. Eleventh Edition, Fourth Indian Reprint. S. Chand & Co. New Delhi.
3. **Anton, Howard, Bivens, Irl, & Davis, Stephen** (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.

Additional Readings:

1. **Dickson, Leonard Eugene** (2009). *First Course in the Theory of Equations*. The Project Gutenberg EBook (<http://www.gutenberg.org/ebooks/29785>).

SEMESTER-VIII

MAT 806G : Numerical Methods

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial ((per week)

Duration: 14 Weeks (70 hrs.) **Examination:** 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students' various topics in Numerical Analysis such as solutions of nonlinear equations in one variable, interpolation and approximation, numerical differentiation and integration, direct methods for solving linear systems, numerical solution of ordinary differential equations.

Course Learning Outcomes: After completion of this course, students will be able to:

- i) Find the consequences of finite precision and the inherent limits of numerical methods.
- ii) Appropriate numerical methods to solve algebraic and transcendental equations.
- iii) Solve first order initial value problems of ODE's numerically using Euler methods.

Unit 1: Errors and Roots of Transcendental and Polynomial Equations

Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

Unit 2: Algebraic Linear Systems and Interpolation

Gaussian elimination method (with row pivoting), Gauss–Jordan method; Iterative methods: Jacobi method, Gauss–Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators, Gregory–Newton forward and backward difference interpolations, Piecewise polynomial interpolation (linear and quadratic).

Unit 3: Numerical Differentiation, Integration and ODE

Numerical differentiation: First and second order derivatives, Richardson extrapolation method; Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equation: Euler's method, Modified Euler's methods (Heun's and midpoint).

References:

1. **Chapra, Steven C.** (2018). *Applied numerical Methods with MATLAB for Engineers and Scientists* (4th ed.). McGraw-Hill Education.
2. **Fausett, Laurene V.** (2009). *Applied Numerical Analysis Using MATLAB*. Pearson. India.
3. **Jain, M. K., Iyengar, S. R. K., & Jain R. K.** (2012). *Numerical Methods for Scientific and Engineering Computation* (6th ed.). New Age International Publishers. Delhi.

Additional Reading:

1. **Bradie, Brian** (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

DISCIPLINE-SPECIFIC ELECTIVE COURSE (DSE)

SEMESTER-V [MAT 701D]

ANY ONE OF THE FOLLOWING

Advanced Group Theory/Mathematical Modeling/Integral Transform

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

MAT 701D (a) : Advanced Group Theory

Course Objectives: The course will develop an in-depth understanding of one of the most important branch of the abstract algebra with applications to practical real-world problems. Classification of all finite Abelian groups (up to isomorphism) can be done.

Course Learning Outcomes: The course shall enable students to learn about:

- i) Automorphisms for constructing new groups from the given group.
- ii) External direct product that applies to data security and electric circuits.
- iii) Group actions, Sylow theorems and their applications to check non-simplicity.

Unit 1: Automorphisms and External and Internal Direct Products

(35 marks, 5 weeks)

Automorphism, inner automorphism, Automorphism groups, Automorphism groups of finite and infinite cyclic groups, Characteristic subgroups, Commutator subgroup and its properties; Applications of factor groups to automorphism groups.

External direct products of groups and its properties, The group of units modulo n as an external direct product, Applications to data security and electric circuits; Internal direct products, Fundamental theorem of finite Abelian groups and its isomorphism classes.

Unit 2: Group Action

(30 marks, 4 weeks)

Group actions and permutation representations; Stabilizers, orbits and kernels of group actions; Groups acting on themselves by left multiplication and consequences; Conjugacy in S_n , Conjugacy classes, The class equation, p -groups.

Unit 3: Sylow Theorems and applications

(35 marks, 5 weeks)

The Sylow theorems and consequences, Applications of Sylow theorems; Classification of groups of order p^2 , where p is a prime, Classification of groups of pq where p, q are distinct primes; Finite simple groups, Non simplicity tests; Generalized Cayley's theorem, Index theorem, Embedding theorem and applications. Simplicity of A_5 .

References:

1. **Dummit, David S., & Foote, Richard M.** (2016). *Abstract Algebra* (3rd ed.). Student Ed. Wiley India.
2. **Gallian, Joseph. A.** (2013). *Contemporary Abstract Algebra* (8th ed.). Cengage Learning India Private Limited. Delhi.
3. **Herstein, I. N.** (2006.) *Topics in Algebra* (2nd Edition). Wiley India.
4. **Michael Artin** (2014). *Algebra* (2nd Edition). Pearson.
5. **Bhattacharya, P.B., Jain, S.K. and Nagpaul, S.R.** (2003). *Basic Abstract Algebra* (2nd Edition). CUP.

Additional Reading:

1. **Rotman, Joseph J.** (1995). *An Introduction to The Theory of Groups* (4th Edn.) Springer-Verlag, New York.
2. **Khanna, Vijay K, Bhambri, S K** (2017). *A Course in Abstract Algebra* (5th edn). Vikas Pub. House.

Teaching Plan (MAT 701D(a): Advanced Group Theory)

Week 1: Automorphism, inner automorphism, Automorphism groups, Automorphism groups of finite and infinite cyclic groups [1] Chapter 4(section 4.4), [2] Chapter 6 (page 135-139)

Week 2: Characteristic subgroups, Commutator subgroup and its properties; Applications of factor groups to automorphism groups, [1] Chapter 4(section 4.4), [2] Chapter 9(Theorem 9.4)

Week 3: External direct products of groups and its properties, The group of units modulo n as an external direct product, Applications to data security and electric circuits, [2] Chapter 8

Week 4: Internal direct products, [2] Chapter 9(section on internal direct product)

Week 5: Fundamental theorem of finite Abelian groups and its isomorphism classes.

[2] Chapter 11, [1]Chapter 5(section 5.2), [5] Chapter 8

Week 6 and 7: Group actions and permutation representations; Stabilizers and kernels of group actions [1] Chapter 1(section 1.7), Chapter 2(section 2.2), Chapter 4(section 4.1)

Week 8: Groups acting on themselves by left multiplication and consequences;

[1] Chapter 4(section 4.2 and 4.3) Conjugacy in S_n

Week 9: Conjugacy classes, The class equation, p -groups, [1] Chapter 4(section 4.3), [5] Chapter 5(section 4)

Week 10, 11 and 12: The Sylow theorems and consequences, Applications of Sylow theorems; Classification of groups of order p^2 , where p is a prime, Classification of groups of pq where p, q are distinct primes

[1] Chapter 4(section 4.5) [2] Chapter 24, Chapter 9(Theorem 9.7), [5] Chapter 8(section 4 and 5)

Week 13 and 14: Finite simple groups, Non simplicity tests; Generalized Cayley's theorem, Index theorem, Embedding theorem and applications. Simplicity of A_5 , [2] Chapter 25,

MAT 701D (b) : Mathematical Modeling

Course Objectives: The main objective of this course is to teach students how to model physical problems using differential equations and solve them. Also, the knowledge of simulation model, linear programming models, and graph theoretic models is imparted by which the listed problems can be solved both numerically and analytically.

Course Learning Outcomes: The course will enable the students to learn the following:

- Know about power series solution of a differential equation and learn about Legendre's and Bessel's equations.
- Learn about various models such as Monte Carlo simulation models, queuing models and linear programming models.
- Understand the basics of graph theory and learn about social networks, Eulerian and Hamiltonian graphs, diagram tracing puzzles and knight's tour problem.

Unit 1: Power Series Solutions

(30 marks, 4 weeks)

Power series solution of a differential equation about an ordinary point, Solution about a regular singular point, the method of Frobenius. Legendre's and Bessel's equation.

Unit 2: Monte Carlo Simulation

(35 marks, 5 weeks)

Monte Carlo Simulation Modeling: Simulating deterministic behavior (area under a curve, volume under a surface); Generating Random Numbers: Middle square method, Linear congruence; Queuing Models: Harbor system, Morning rush hour. Overview of optimization modeling; Linear Programming Model: Geometric solution, Algebraic solution, Simplex method, Sensitivity analysis.

Unit 3: Graph Theory

(35 marks, 5 weeks)

Graphs, Diagraphs, Networks and subgraphs, Vertex degree, Paths and cycles, Regular and bipartite graphs, Four cube problem, Social networks, Exploring and traveling, Eulerian and Hamiltonian graphs, Applications to dominoes, Diagram tracing puzzles, Knight's tour problem, Gray codes.

References:

- Aldous, Joan M., & Wilson, Robin J.** (2007). *Graphs and Applications: An Introductory Approach*. Springer. Indian Reprint.
- Edwards, C. Henry, Penney, David E., & Calvis, David T.** (2015). *Differential Equation and Boundary Value Problems: Computing and Modeling* (5th ed.). Pearson.
- Giordano, Frank R., Fox, William P., & Horton, Steven B.** (2014). *A First Course in Mathematical Modeling* (5th ed.). Brooks/Cole, Cengage Learning.

Teaching Plan (MAT 701D (b): Mathematical Modeling)

Weeks 1, 2 and 3: Power series solution of a differential equation about an ordinary point, Solution about a regular singular point. Legendre's equation. The method of Frobenius. [2] Chapter 8 (Sections 8.1 to 8.3).

Week 4: Bessel's equation. Bessel's function of first kind. [2] Chapter 8 [Section 8.5 up to Page 553].

Weeks 5, 6 and 7: Monte Carlo simulation modeling: Simulating deterministic behavior (area under a curve, volume under a surface), Generating random numbers: Middle square method, Linear congruence. Queuing models: Harbor system, Morning rush hour. [3] Chapter 5 (Sections 5.1 to 5.2, and 5.5).

Weeks 8 and 9: Overview of optimization modeling, Linear programming model: Geometric solution, Algebraic solution, Simplex method, Sensitivity analysis. [3] Chapter 7.

Weeks 10, 11 and 12: Graphs, Diagraphs, Networks and subgraphs, Vertex degree, Paths and cycles, Regular and bipartite graphs, Four cube problem, Social networks. [1] Chapter 1 (Section 1.1), and Chapter 2.

Weeks 13 and 14: Exploring and traveling, Eulerian and Hamiltonian graphs, Applications to dominoes, Diagram tracing puzzles, Knight's tour problem, Gray codes. [1] Chapter 3.

MAT 701D (c) : Integral Transforms

Course Objective: The main objectives of this course are to teach students to form and solve Fourier series, Laplace transforms and Fourier transforms and use them in solving some physical problems.

Course Learning Outcomes: This course will enable the students to:

- i) Learn Fourier series, Euler's formulae, Bessel's inequality, Fourier series in complex form.
- ii) Know about piecewise continuous functions, Dirac's delta function, Laplace transforms and its properties.
- iii) Solve ordinary differential equations using Laplace transforms.
- iv) Familiarise with Fourier transforms of functions belonging to class A, relation between Laplace and Fourier transforms.
- v) Explain Parseval's identity, Plancherel's theorem and applications of Fourier transforms to boundary value problems.

Unit 1: Fourier Series

(25 marks, 3 weeks)

Fourier series, Euler's formulae, Dirichlet's conditions for Fourier series, Fourier sine and cosine series, Convergence of Fourier series, Fourier series for even and odd functions, Half-Range Fourier series, Half-Range expansions, Bessel's inequality, The complex form of Fourier series.

Unit 2: Laplace Transforms

(40 marks, 6 weeks)

Integral transform, Kernel of an integral transform, Laplace transform, Existence theorem, Linearity property, Shifting theorems, Change of scale property, Laplace transforms of derivatives and integrals, Laplace transforms of periodic functions, Integral equations, Dirac's delta function.

Further properties of Laplace transforms: Multiplication by positive integral power of t , Division by t , Inverse Laplace transform, Lerch's theorem, Linearity property of inverse Laplace transform, Inverse transform of derivatives, Convolution theorem, Integral equations, Applications of Laplace transform in obtaining solutions of ordinary differential equations and integral equations.

Unit 3: Fourier Transforms**(35 marks, 5 weeks)**

Fourier and inverse Fourier transforms, Fourier sine and cosine transforms, Inverse Fourier sine and cosine transforms, Relation between Fourier and Laplace transforms, Linearity property, Change of scale property, Shifting property, Modulation theorem, Solution of integral equation by Fourier sine and cosine transforms, Convolution theorem for Fourier transform, Parseval's identity for Fourier transform, Plancherel's theorem, Fourier transform of derivatives, Applications of infinite Fourier transforms to boundary value problems, Finite Fourier transform, Inversion formula for finite Fourier transforms.

References:

1. **James Ward Brown & Ruel V. Churchill** (2011), *Fourier Series and Boundary Value Problems*. McGraw-Hill Education.
2. **Dr. S. Sreenadh, S. Ranganathan, Dr. M. V. S. S. N. Prasad & Dr. V. Ramesh Babu**, *Fourier series & Integral transforms* (Reprint 2020), S Chand.
3. **MD Raisinghania** (2022), *Advanced Differential Equations* (20th edition), S Chand and Company Ltd., New Delhi.

Additional Reading:

1. **Erwin Kreyszig** (2011). *Advanced Engineering Mathematics* (10th edition). Wiley
2. **A. Zygmund** (2002). *Trigonometric Series* (3rd edition). Cambridge University Press.
3. **Phil Dyke** (2014). *An Introduction to Laplace Transform and Fourier*. Springer.
4. **Rajendra Bhatia**. *Fourier Series* (2nd edition). Hindustan Book Agency (India), New Delhi.
5. **H K Dass** (2018). *Advanced Engineering Mathematics* (22nd edition). S Chand and Company Ltd., New Delhi.

Teaching Plan (MAT 701D(C): Integral Transforms):

Week 1: Fourier series, Periodic function, Euler's formulae, Dirichlet's Conditions for Fourier series, Convergence of Fourier series. [3] [Part-III A, Ch-1 (Section 1.1- 1.4)]

Week 2: Fourier series for even and odd functions, Half- Range Fourier sine and cosine series. Half-Range expansions. [2] [Ch-1 (Section 1.6 to 1.11)]

Week 3: Bessel's inequality & the complex form of Fourier series. [2] [Ch-4 Section 4.1, [1] Ch-2 Section 9-12]

Week 4: Integral transform, Kernel of an integral transform, Laplace transform, Existence theorem, Linearity property. [3] [Part-IV A, Ch-1 (Section 1.1-1.8)]

Week 5: Shifting theorems, Change of scale property, Laplace transforms of derivatives and integrals, Further properties of Laplace transforms: Multiplication by positive integral power of t, Division by t. [3] [Part-IV A, Ch-1 (Section 1.11, 1.13 to 1.18)]

Week 6: Laplace transform of periodic functions, Integral equations & Dirac's delta function. [3] [Part-IV A, Ch-1 (Section 1.20 to 1.22 (II))]

Week 7: Inverse Laplace transform, Lerch's theorem, Linearity property of inverse Laplace transform. [3] Part-IV A, [Ch-2 (Section 2.2, 2.3A & 2.5)]

Week 8: Inverse transform of derivatives and integrals, Convolution theorem. [3] [Part-IV A, Ch-2 (Section 2.11, 2.12, & 2.16)]

Week 9: Applications of Laplace transform in obtaining solutions of ordinary differential equations and integral equations.[3] [Part-IVA, Ch-3 & 4]

Week 10: Fourier and inverse Fourier transforms, Fourier sine and cosine transforms, Inverse Fourier sine and cosine transforms, Relation between Fourier and Laplace transforms.[3] [Part-IVB, Ch-1 (Section 1.6 to 1.9)]

Week 11: Linearity property, Change of scale property, Shifting property, Modulation theorem, Solution of the integral equation by Fourier sine and cosine transforms.[3] [Part-IVB, Ch-1 (Section 1.11 to 1.14 & 1.17)]

Week 12: Convolution theorem for Fourier transform, Parseval's identity for Fourier transforms, Plancherel's theorem, Fourier transform of derivatives. [3] [Part-IVB, Ch-1 (Section 1.20 to 1.22)]

Week 13: Applications of infinite Fourier transforms to boundary value problems. [3] [Part-IVB, Ch-1 Sec III,]

Week 14: Finite Fourier transform, Inversion formula for finite Fourier transforms.[3] [Part-IVB, Ch-2 (Section I & II)]

SEMESTER-VI [MAT 702D]

ANY ONE OF THE FOLLOWING

Special Theory of Relativity & Tensors/ Linear Programming and its Applications/ Probability Theory and Statistics

MAT 702D (a) : Special Theory of Relativity & Tensors

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The course aims at providing the basic knowledge of Newtonian mechanics, Relativistic kinematics, Relativistic mechanics and Tensor calculus.

Course Learning Outcomes: The course will enable the students to:

- i) Understand the basic elements of Newtonian mechanics including Michelson-Morley experiment and geometrical interpretations of Lorentz transformation equations.
- ii) Learn about length contraction, time dilation and Lorentz contraction factor.
- iii) Study 4-dimensional Minkowskian space-time and its consequences.
- iv) Learn about the transformation of co-ordinates, contravariant and covariant tensors.

- v) Understand the algebraic operations of tensors, symmetric and skew-symmetric tensors

Unit 1: Newtonian Mechanics and Relativistic Kinematics

(35 marks, 5 weeks)

Inertial frames, Galilean transformation, Michelson-Morley experiment, Lorentz-Fitzgerald contraction hypothesis, Relativistic concept of space and time, Postulates of special theory of relativity.

Lorentz transformation equations and its geometrical equations, Group properties and its geometrical equations, consequences of Lorentz transformation equations like Relativity of Simultaneity, Einstein's time dilation, length contraction and related problems, transformation equations for components of velocity and acceleration of a particle.

Unit 2: Relativistic Mechanics

(35 marks, 5 weeks)

Variation of mass with velocity, Equivalence of mass and energy and its consequences, Transformation equations for mass, momentum, energy and force. Relation between momentum and energy. Energy momentum four-vector.

Four-dimensional Minkowskian spacetime of special relativity, time-like, light-like and space-like intervals, Null cone, proper time, world line of a particle, Four tensors in Minkowskian space-time.

Unit 3: Tensors

(30 Marks, 4 weeks)

Space of N-dimension, Transformation of co-ordinates, contravariant and covariant vectors (Tensor of first order), Tensor of second order (or of rank two), Tensors of higher rank (or higher orders), Mixed tensors, Kronecker delta symbol, Invariant or scalar, Algebraic operations with tensors, Addition & subtraction of tensors, contraction, product of tensors, Inner Product, symmetric and Skew symmetric tensor.

References:

1. **Farook Rahaman (2014):** *The Special Theory of Relativity, A Mathematical Approach*, Springer
2. **Robert Resnick (2007):** *Introduction to Special Relativity*, John Wiley
3. **James L Anderson (1973):** *Introduction to the Theory of Relativity*, Dover publications

Additional Reading:

1. **M. Ray:** *Special Theory of Relativity*, S Chand and Co
2. **A. Das (1993):** *The Special Theory of Relativity*, Springer
3. **Banerjee and Banerjee (2012):** *The Special Theory of Relativity*, PHI, New Delhi.
4. **Dirac :** *General Theory of Relativity*, Prentice Hall of India, New Delhi.
5. **S.K. Bose:** *General Theory of Relativity*, Wiley Eastern Ltd.

Teaching plan (MAT 702D (a) : Special Theory of Relativity and tensors).

Week 1: Inertial frames, Galilean transformation[1] chapter 1 (section 1.1, 1.2,1.3) Michelson-Morley experiment [1] chapter 2 (section 2.2) Lorentz-Fitzgerald contraction hypothesis [1] chapter2 (section2.4)

Week 2: Relativistic concept of space & time[1] chapter 2 (section2.5)

Postulates of Special Theory of Relativity [1] chapter 3 (section3.1)

Week 3: Lorentz transformation equations and its geometrical equations[1] chapter 3 {section(3.2) 3.2.1} Group properties of Lorentz transformation.

Week 4: Consequences of Lorentz equations like Relativity of Simultaneity[1] chapter 4 (section4.3), Einstein's time dilation[1] chapter 4 (section4.2) length contraction[1] chapter 4 (section 4.1)

Week 5: Transformation equations for components of velocity [1] chapter 7 (section7.1, section 7.2), acceleration of a particle[1] chapter 7 (section 7.3, section 7.4)

Week 6: Variation of mass with velocity [2] chapter 3 (section 3.3) Equivalence of mass and energy and its consequences [1] chapter 10 (section 10.4)

Week 7: Transformation equations for mass, momentum, energy and force [2] chapter 3 (section 3.7)

Week 8: Relation between momentum and energy [1] chapter 10 (section 10.5, section 11.11) Energy momentum four vector [1] chapter 10 (10.2)

Week 9: Four dimensional Minkowskian space-time of special relativity, time-like, light-like and space like intervals, Null cone [1] chapter 8 (section 8.1)

Week 10: Proper time [1] chapter 8 (section 8.2) World line of a particle, Four vectors in Minkowskian space-time [1] chapter 8 (section 8.4)

MAT 702D (b) : Linear Programming and its Applications

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: This course develops the ideas underlying the Simplex Method for Linear Programming Problem, as an important branch of Operations Research. The course covers Linear Programming with applications to Transportation, Assignment and Game Problem. Such problems arise in manufacturing resource planning and financial sectors.

Course Learning Outcomes: This course will enable the students to learn:

- i) Analyze and solve linear programming models of real-life situations.
- ii) The graphical solution of LPP with only two variables, and illustrate the concept of convex set and extreme points. The theory of the simplex method is developed.
- iii) The relationships between the primal and dual problems and their solutions with applications to transportation, assignment and two-person zero-sum game problem.

Unit 1: Introduction to Linear Programming (30 marks, 4 weeks)

The Linear Programming Problem: Standard, Canonical and matrix forms, Graphical solution. Convex and polyhedral sets, Hyperplanes, Extreme points. Basic solutions; Basic Feasible Solutions; Reduction of any feasible solution to a basic feasible solution; Correspondence between basic feasible solutions and extreme points.

Unit 2: Methods of Solving Linear Programming Problem and dual problem (40 marks, 6 weeks)

Simplex Method: Algebra of Simplex method, Optimal solution, Termination criteria for optimal solution of the Linear Programming Problem, Unique and alternate optimal solutions, Unboundedness; Simplex Algorithm and its Tableau Format; Artificial variables, Two-phase method, Big-M method.

Motivation and Formulation of Dual problem; Primal-Dual relationships; Fundamental Theorem of Duality; Complimentary Slackness.

Unit 3: Applications (30 marks, 4 weeks)

Transportation Problem: Definition and formulation; Methods of finding initial basic feasible solutions; North West corner rule. Least cost method; Vogel's Approximation method; Algorithm for solving Transportation Problem.

Assignment Problem: Mathematical formulation and Hungarian method of solving.

Game Theory: Basic concept, Formulation and solution of two-person zero-sum games, Games with mixed strategies, Linear Programming method of solving a game.

References:

1. **Bazaraa, Mokhtar S., Jarvis, John J., & Sherali, Hanif D.** (2010). *Linear Programming and Network Flows* (4th ed.). John Wiley and Sons.
2. **Hadley, G.** (1997). *Linear Programming*. Narosa Publishing House. New Delhi.
3. **Taha, Hamdy A.** (2010). *Operations Research: An Introduction* (9th ed.). Pearson.

Additional Readings:

1. **Hillier, F. S. & Lieberman, G. J.** (2010). *Introduction to Operations Research- Concepts and Cases* (9th ed.). Tata McGraw Hill.
2. **Thie, Paul R., & Keough, G. E.** (2014). *An Introduction to Linear Programming and Game Theory*. (3rd ed.). Wiley India Pvt. Ltd.

Teaching Plan (MAT 702D(b): Linear Programming and its Applications)

Week 1: The Linear Programming Problem: Standard, Canonical and matrix forms, Graphical solution.

[1] Chapter 1(section 1.1) [2] Chapter 1 (sections 1.1 to 1.4 and 1.6)

Week 2&3: Convex and polyhedral sets, Hyperplanes, Extreme points. Basic Solutions; Basic Feasible Solutions.

[1] Chapter 2 (section 2.4 to 2.7) Chapter 3(section 3.2), [2] Chapter 2(section 2.16, 2.19, 2.20)

Week 4: Reduction of any feasible solution to a basic feasible solution; Correspondence between basic feasible solutions and extreme points.

[1] Chapter 3(section 3.2), [2] Chapter 3(section 3.4, 3.5, 3.6, 3.10)

Week 5: Simplex Method: Algebra of Simplex method, Optimal solution, Termination criteria for optimal solution of the Linear Programming Problem, Unique and alternate optimal solutions, Unboundedness.

[1] Chapter 3(section 3.3, 3.5 3.6), [2] Chapter 3(section 3.7, 3.8, 3.9)

Week 6 : Simplex Algorithm and its Tableau Format.

[1] Chapter 3(section 3.7, 3.8) [2] Chapter 4(section 4.8, to 4.11)

Week 7: Artificial variables, Two-phase method, Big-M method.

[1] Chapter 4(section 4.1 to 4.3) [2] Chapter 5(section 5.1 to 5.5)

Week 8 to 10: Motivation and Formulation of Dual problem; Primal-Dual relationships; Fundamental Theorem of Duality; Complimentary Slackness theorem with examples.

[1] Chapter 6(section 6.1, 6.2) [2] Chapter 8(section 8.1 to 8.5)

Week 11 to 12: Transportation problem, Assignment Problem

[3] Chapter 5(section 5.1, 5.3, 5.4)

Week 13 to 14: *Game Theory:* Basic concept, Formulation and solution of two-person zero-sum games, Games with mixed strategies, Linear Programming method of solving a game.

[3] Chapter 11(section 11.12, 11.13)

MAT 702D (c) : Probability Theory and Statistics

Total Marks: 100 (Theory: 75 + Internal Assessment: 25)

Workload: 5 Lectures (per week), 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: To make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness. The course intends to render the students to several examples and exercises that blend their everyday experiences with their scientific interests.

Course Learning Outcomes: This course will enable the students to learn:

- i) Distributions to study the joint behavior of two random variables.

- ii) To establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.
- iii) Central limit theorem, which helps to understand the remarkable fact that: the empirical frequencies of so many natural populations, exhibit a bell-shaped curve.

Unit 1: Probability Functions and Moment Generating Function (30 marks, 4 weeks)

Sample space, Probability set function, Real random variables – Discrete and continuous, Cumulative distribution function, Probability mass/density functions, Transformations, Mathematical expectation, Moments, Moment generating function, Characteristic function.

Unit 2: Univariate Discrete, Continuous Distributions and Bivariate distributions (35 marks, 5 weeks)

Discrete distributions: Uniform, Bernoulli, Binomial, Negative binomial, Geometric and Poisson;

Continuous distributions: Uniform, Gamma, Exponential, Chi-square, Beta and Normal; Normal approximation to the binomial distribution.

Bivariate Distribution: Joint cumulative distribution function and its properties, Joint probability density function, Marginal distributions, Expectation of function of two random variables, Joint moment generating function, Conditional distributions and expectations.

Unit 3: Correlation, Regression and Central Limit Theorem (35 marks, 5 weeks)

The Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function, Independent random variables, Linear regression for two variables, The method of least squares, Bivariate normal distribution, Chebyshev's theorem, Strong law of large numbers, Central limit theorem and weak law of large numbers.

References:

1. Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2013). *Introduction to Mathematical Statistics* (7th ed.). Pearson Education, Inc.
2. Miller, Irwin & Miller, Marylees. (2014). John E. Freund's *Mathematical Statistics with Applications* (8th ed.). Pearson. Dorling Kindersley (India).

Additional Reading:

1. Ross, Sheldon M. (2014). *Introduction to Probability Models* (11th ed.). Elsevier Inc. AP.
2. Mood, A.M., Graybill, F.A. & Boes, D.C. (1974). *Introduction to the Theory of Statistics* (3rd ed.). McGraw-Hill Education Pvt. Ltd. Indian edition (2017).

Teaching Plan (MAT 702D (c): Probability Theory and Statistics)

Week 1: Sample space, Probability set function, Real random variables – Discrete and continuous. [1] [Chapter-1(Section 1.1, 1.3, 1.5 to 1.7).

Week 2: Cumulative distribution function, Probability mass/density functions. [2] Chapter -2 Section 2 & 4).

Week 3: Transformations, Mathematical expectation. [1] Ch-1 (Section 1.7.2&1.8).

Week 4: Moments, Moment generating function, Characteristic function. [1] Chapter-1 (Section 1.9).

Week 5: Discrete distributions: Uniform, Bernoulli, Binomial, Negative binomial, Geometric and Poisson. [2] Chapter-5 (Section 2 to 5 & 7).

Week 6: Continuous distributions: Uniform, Gamma, Exponential, Chi-square, Beta and Normal; Normal approximation to the binomial distribution. [2] Chapter-6 (Section 2 to 6).

Week 7: Bivariate Distribution: Joint cumulative distribution function and its properties, Joint probability density function. [1] Chapter-2 (Section 2.1).

Week 8: Marginal distributions, Expectation of function of two random variables, Joint moment generating function. [1] Chapter-2 (Section 2.1.1, 2.1.2).

Week 9: Conditional distributions and expectations. [1] Chapter-2 (Section 2.3).

Week 10: The Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function. [1] Chapter -2 (Section 2.5, 2.5.1).

Week 11: Independent random variables, Linear regression for two variables.
[1] Chapter-2 (Section 2.4) & [2] Ch-6 (Section 2).

Week 12: The method of least squares, Bivariate normal distribution.
[2] Chapter-6 (Section 3) & [2] Ch-5 (Section 7).

Week 13: Chebyshev's theorem, Strong law of large numbers. [2] Chapter-3 (Section 4).

Week 14: Central limit theorem and weak law of large numbers. [1] Chapter-5 (Section 5.3, 5.4).

SEMESTER-VII [MAT 803D]

ANY ONE OF THE FOLLOWING

Advanced Complex Analysis/Functional Analysis/Graph Theory

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week) 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

MAT 803D (a) : Advanced Complex Analysis

Course Objectives: This course aims to introduce the basic ideas of analysis for complex integration and meromorphic functions as well as bilinear transformations.

Course Learning Outcomes

- i). To get equipped with the understanding of the fundamental concepts of functions of a complex variable along with the concepts of analyticity, Cauchy-Riemann relations and harmonic functions.
- ii). Evaluate complex contour integrals applying the Cauchy's integral theorem and Cauchy's integral formula.
- iii). To acquire the skill of evaluating contour integrals using Cauchy's integral formula and Cauchy's integral theorem.

Unit – 1: Complex integration: Cauchy-Goursat Theorem for simple closed contour, simply and multiply connected domain; Cauchy's integral formula and higher order derivatives, Morera's theorem; Maximum modulus principle, Schwarz lemma; Taylor's theorem, Laurent's theorem, Counting zeros; the open mapping theorem.

Unit – 2: Isolated singularities: Residues and poles, Cauchy's residue theorem, zeros of analytic functions, Evaluation of improper integrals, Meromorphic functions; The argument principle, Rouch's theorem, Inverse Laplace Transform and examples.

Unit – 3: Mappings by elementary functions: Branches of many valued functions with special reference to $\arg z$, $\log z$ and z^n , Definitions and examples of Conformal mapping, Bilinear transformations, their properties and classifications. Basic properties of Harmonic functions, Harmonic functions on a disk, Harnack's inequality; Jensen's formula, Poisson-Jensen formula, Canonical products, Hadamard's three circles theorem.

References:

1. **J.B. Conway (2002)**, *Functions of One Complex Variable*, Narosa Publishing House.
2. **H.A. Priestly (1990)**, *Introduction to Complex Analysis*, Clarendon Press, Oxford.

Additional reading :

1. **James W Brown, R V Churchill (2004)**, *Complex variables and Applications*, McGraw Hill.
2. **Liang-Shin Hahn and Bernard Epstein (1996)**, *Classical Complex Analysis*, Jones and Bertlett Pub. International, London.
3. **Walter Rudin (1987)**, *Real & Complex Analysis*, 3rd Edn. McGraw-Hill Book Co.
4. **S. Ponnusamy (1997)**, *Foundation of Complex Analysis*, Narosa Pub. House.
5. **H.S. Kasana (2005)**, *Complex Variables; Theory and Applications*, Prentice-Hall of India, New Delhi.

MAT 803D (b) : Functional Analysis

Course Objectives: This course aims at imparting concepts on a normed space in particular Banach and Hilbert spaces. The equivalence of various norms and various results on Banach spaces will be studied.

Course Learning Outcomes :

After completion of this course, the students may be able

- i). To learn the concept of a norm, the conditions for a norm space to be a Banach space, the quotient space of a normed space and its completeness with respect to a norm, norms to be equivalent, compactness of finite dimensional normed spaces, bounded linear transformations and dual space.
- ii). To learn Uniform Boundedness Theorem with applications to Open mapping Theorem and Closed graph Theorem, Hahn-Banach Theorem in normed spaces, reflexive space.
- iii). To learn inner product space, Hilbert space, orthonormal and complete orthonormal sets of an inner product space, Bessel's inequality and Parseval's identity, explore the structure of Hilbert spaces, learn projection Theorem and Riesz representation theorem.

Unit – 1: Normed linear spaces, Banach spaces and examples, Quotient space of normed linear spaces and its completeness, equivalent norms, Riesz Lemma, basis properties of finite dimensional normed linear spaces and compactness, Weak convergence and bounded linear transformations, normed linear spaces of bounded linear transformations, dual spaces with examples.

Unit – 2: Uniform boundedness theorem and some of its consequences, Open mapping and closed graph theorems, Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces, Reflexive spaces.

Unit – 3: Inner product spaces, Hilbert spaces, Orthonormal Set, Bessel's inequality, Complete orthonormal sets and Parseval's identity, Structure of Hilbert spaces, Projection theorem, Riesz representation theorem.

References:

1. **D. Kreyszig** (1978), *Introductory Functional Analysis with Applications*, John Wiley & Sons, New York, 1978.
2. **H.L. Royden** (1993), *Real Analysis*, Macmillan Pub. Co. Inc. New York, 4th Edition.

Additional reading :

1. **C. Goffman and G. Pedrick** (1987): *First Course in Functional Analysis*, Prentice Hall of India, New Delhi.
2. **P.K. Jain, O.P. Ahuja and Khalil Ahmad** (1997), *Functional Analysis*, New Age International (P) Ltd. & Wiley Eastern Ltd. New Delhi.
3. **R.B. Holmes** (1975), *Geometric Functional Analysis and its Applications*, Springer-Verlag.
4. **K.K. Jha** (1982), *Functional Analysis*, Students' Friends.
5. **L.V. Kantorovich and G.P. Akilov** (1982), *Functional Analysis*, Pergamon Press.

MAT 803D (c) : Graph Theory

Course Objectives: This course aims to introduce the basic ideas of graph theory with applications to Networks. This course will enhance the problem solving skills and research related skills.

Course Learning Outcomes: This course will enable the students to:

- i). Appreciate the definition and basics of graphs along with types and their examples.
- ii). Understand the definition of a tree and learn its applications to fundamental circuits.
- iii). Know the applications of graph theory to network flows.
- iv). Understand the notion of planarity and coloring of a graph.
- v). Relate the graph theory to the real-world problems

Unit-1: Paths, Circuits and Graph Isomorphisms

Definition and examples of a graph, Subgraph, Walks, Paths and circuits; Connected graphs, disconnected graphs and components of a graph; Euler and Hamiltonian graphs, Graph isomorphisms, Adjacency matrix and incidence matrix of a graph, Directed graphs and their elementary properties.

Unit-2: Trees and Fundamental Circuits

Definition and properties of trees, Rooted and binary trees, Cayley's theorem on a counting tree, Spanning tree, Fundamental circuits, Minimal spanning trees in a connected graph.

Cut-set of a graph and its properties, Fundamental circuits and cut-sets, Cut-vertices, Connectivity and separability, Network flows, 1- isomorphism and 2- isomorphism.

Unit-3: Planar Graphs and Coloring

Planar graph, Euler theorem for a planar graph, Various representations of a planar graph, Dual of a planar graph, Detection of planarity, Kuratowski's theorem. Chromatic number of a graph, Chromatic partition, Chromatic polynomial, Matching and coverings, Four color problem.

References:

1. **R. Balakrishnan & K. Ranganathan** (2012). *A Textbook of Graph Theory*. Springer.
2. **C.L. Liu**, *Elements of Discrete Mathematics*, Mc-Graw-Hill Book Co.

Additional reading :

1. **N. Deo** (2016), *Graph Theory with Applications to Engineering and Computer Sciences*, Dover Publication.
2. **F Harary** (1969), *Graph Theory*, Addison Wesley Publishing Company Reprinted Narosa Publishing House
3. **J.P. Tremblay & R. Manohar** (1997), *Discrete Mathematical Structures with Applications to Computer Science*, McGraw-Hill Book Co.
4. **J.L. Gersing** (2014), *Mathematical Structures for Computer Science*, W H Freeman nad Co.
5. **Seymour Lipschutz** (1983), *Finite Mathematics*, McGraw-Hill Book Company, New York.
6. **S. Wiitala** (1987), *Discrete Mathematics – A Unified Approach*, McGraw-Hill Book Co.
7. **Kolman, Busby, Ross** (2004), *Discrete Mathematical Structures*, Prentice Hall of India (reprint)

SEMESTER-VIII [MAT 804D]

ANY ONE OF THE FOLLOWING
Advanced Partial Differential Equations / Fixed Point Theory /
Cryptography

MAT 804D (a) : Advanced Partial Differential Equations

Total Marks: 100 (Theory: 75, Internal Assessment: 25)

Workload: 5 Lectures (per week) 1 Tutorial (per week)

Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: This course aims to give knowledge of PDEs which are used to mathematically formulate physical and other problems involving functions of several variables, such as the propagation of heat or sound, fluid flow, elasticity, electrostatics, electrodynamics such as propagation of light and sound in the atmosphere, and of waves on the surface of pond.

Course Learning Outcomes: After studying the course the students may have a sound knowledge of

- i). Understanding of the real-valued convex functions of one real variable, functions of one quantity such as position, pressure, or temperature, momentum, volume, and entropy .
- ii).The application in image analysis, image filtering, image reconstruction and image compression, Linear differential transformation equations into algebraic equations, tunneling of bosons, solutions of different partial differential equation .

Unit – 1: Examples of PDE. Classification, Nonlinear First Order PDE – Complete Integrals, Envelopes, Characteristics, Hamilton-Jacobi Equations (Calculus of Variations, Hamilton's ODE, Legendre Transform, Hopt-Lax Formula, Weak Solutions, Uniqueness), Conservation Laws (Lax-Oleinik formula, Weak Solutions).

Unit – 2: Representation of Solutions – Separation of Variables, Similarity Solutions (Plane and Travelling Waves, Similarity under Scaling), Fourier and Laplace Transform, Hopf-Cole Transform, Hodograph and Legendre Transforms, Potential Functions, Power Series (Non-characteristics Surfaces, Real Analytic Functions, Cauchy-Kovalevskaya Theorem).

Unit – 3: Transport Equation – Initial value Problem. Non-homogeneous Equation.

Laplace's Equation – Fundamental Solution, Mean and Formulas, Properties of Harmonic Functions, Green's Function, Energy Methods.

Heat Equation – Fundamental Solution, Mean Value Formulation, Properties of Solutions, Energy Methods.

Wave Equation – Solution by Spherical Means, Non-homogeneous Equations, Energy Methods.

References:

1. **L.C. Evans** (1998), *Partial Differential Equations*, Graduate Studies in Mathematics, Volume 19, AMS.

Additional reading:

1. **I.N. Sneddon** (2006), *Elements of Partial Differential Equations*, Dover Publications.
2. **F. John** (1991). *Partial Differential Equations*, Springer
3. **P. Prasad, R. Ravindran** (1984). *Partial Differential Equations*, John Wiley.
4. **I.M. Gelfand and S.V. Fomin**, *Calculus of Variations*, Prentice Hall.
5. <http://archive.nptel.ac.in/courses/111/103/111103021/>

MAT 804D (b) : Fixed Point Theory

Course Objectives: This course aims at imparting the problem solving and research related skills using the concepts of contraction principles and fixed point theory.

Course Learning Outcomes

After studying the course, students will understand

- i). The concept of metric and Banach spaces and their properties.
- ii). Contraction principle, its converse and applications.
- iii). Metric fixed point theory, Banach fixed point theorem, its generalisations and applications.

Unit 1: Review the concept of metric space and Banach space-definition and properties of metric space, complete metric space, definitions and properties of normed and Banach spaces.

Unit 2: Contraction Principle-Introduction, principle of contraction mapping, linear operators, generalization of contraction mappings, approximate iteration, converse of the contraction principle, applications of contraction principle.

Unit 3: Fixed point theory in metric spaces-introduction of fixed point theory, Banach fixed point theorem, applications of the Banach fixed point theorem, generalizations of the Banach fixed point theorem.

References:

1. **M. A. Khamsi and W. A. Kirk** (2001), *An introduction to Metric Spaces and Fixed Point Theory*, Wiley-Inter Sci., New York.
2. **W. A. Kirk and B. Sims** (2001), *Hand Book of Metric Fixed Point Theory*, Springer, Netherlands, 2001.

Additional reading:

1. **K. C. Border**, *Fixed point theorems with applications to economics and game theory*, Cambridge University Press, Cambridge, 1985.
2. **S. Singh, B. Watson and P. Srivastava**, *Fixed Point Theory and Best Approximation: The KKM - map Principle*, Kluwer Academic Publishers, Dordrecht, 1997.
3. **M. C. Joshi and R. K. Bose**, *Some topics in nonlinear functional analysis*, Wiley Eastern Ltd, 1985.
4. **Erwin Kreyszig**, *Introductory Functional Analysis with Applications*, John Wiley & Sons, NY, 1978.
5. **Vasile I. Istratescu**, *Fixed point Theory (An Introduction)*, D. Reidel Publishing Co., 1981.

MAT 804D (c) : Cryptography

Course Objectives: This course aims at providing knowledge of the important aspects and application of cryptography and aspects of number theory in the present times.

Course Learning Outcomes: This course will enable the students to:

- i). Understand the difference between classical and modern cryptography.
- ii). Learn the fundamentals of cryptography, including Data and Advanced Encryption Standards (DES & AES) and RSA.
- iii). Encrypt and decrypt messages using block ciphers, sign and verify messages using well-known signature generation and verification algorithms.
- iv). Know about the aspects of number theory which are relevant to cryptography.

Unit 1: Introduction to Cryptography and Classical Cryptography

Cryptosystems and basic cryptographic tools: Secret-key cryptosystems, Public-key cryptosystems, Block and stream ciphers, Hybrid cryptography, Message integrity: Message authentication codes, Signature schemes, Nonrepudiation, Certificates, Hash functions,

Cryptographic protocols, Security; Hybrid cryptography: Message integrity, Cryptographic protocols, Security, Some simple cryptosystems, Shift cipher, Substitution cipher, Affine cipher, Vigenère cipher, Hill cipher, Permutation cipher, Stream ciphers, Cryptanalysis of affine, substitution, Vigenère, Hill and LFSR stream ciphers.

Unit-2: Cryptographic Security, Pseudo Randomness and Symmetric Key Ciphers

Shannon's theory, Perfect secrecy, Entropy, Spurious keys and unicity distance; Bit generators, Security of pseudorandom bit generators. Substitution-permutation networks, Data encryption standard (DES), Description and analysis of DES; Advanced encryption standard (AES), Description and analysis of AES; Stream ciphers, Trivium.

Unit-3: Basics of Number Theory and Public-Key Cryptography

Basics of number theory; Introduction to public-key cryptography, RSA cryptosystem, Implementing RSA; Primality testing, Legendre and Jacobi symbols, Solovay-Strassen algorithm, Miller-Rabin algorithm; Square roots modulo n , Factoring algorithms, Pollard $p - 1$ algorithm, Pollard rho algorithm, Dixon's random squares algorithm, Factoring algorithms in practice; Rabin cryptosystem and its security. Basics of finite fields; ElGamal cryptosystem, Algorithms for the discrete logarithm problem, Shanks' algorithm, Pollard rho discrete logarithm algorithm, Pohlig-Hellman algorithm; Discrete logarithm algorithms in practice, Security of ElGamal systems, Bit security of discrete logarithms.

References:

1. **Jeffrey Hoffstein, Jill Pipher & Joseph H. Silverman** (2014). *An Introduction to Mathematical Cryptography* (2nd edition). Springer.

Additional reading:

1. **Neal Koblitz** (1994). *A Course in Number Theory and Cryptography* (2nd edn). Springer-Verlag.
2. **Christof Paar & Jan Pelzl** (2014). *Understanding Cryptography*. Springer.
3. **Simon Rubinstein-Salzedo** (2018). *Cryptography*. Springer.
4. **Douglas R. Stinson & Maura B. Paterson** (2019). *Cryptography Theory and Practice* (4th edition). Chapman & Hall/CRC Press, Taylor & Francis.